HYDROGEN ATOM: PROBABILITY OF FINDING ELECTRON INSIDE THE NUCLEUS

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 4.45.

One rather frivolous calculation that we can do with the hydrogen atom is to figure out the probability of the electron being found inside the nucleus (proton). For this, we need a size for the proton, and an approximate value is $b \approx 10^{-15}$m.

For the ground state, the probability that $r \le b$ is

$$Prob(r \le b) = \frac{1}{\pi a^3}\int_0^{2\pi}\int_0^{\pi}\int_0^{b} e^{-2r/a} r^2 \sin\theta \, dr d\theta d\phi \quad (1)$$

$$= 1 - e^{-2b/a}\left(1 + \frac{2b}{a} + \frac{2b^2}{a^2}\right) \quad (2)$$

Given that the Bohr radius is around 0.5×10^{-10} m, it is difficult to do the calculation with the exponential using such a small exponent, so we can try an approximation. Expanding in a Taylor series using the variable $\varepsilon \equiv 2b/a$ gives

$$Prob(r \le b) = \frac{1}{6}\varepsilon^3 - \frac{1}{8}\varepsilon^4 + \frac{1}{20}\varepsilon^5 + \mathcal{O}(\varepsilon^6) \quad (3)$$

The lowest order term is thus $\varepsilon^3/6 = 4b^3/3a^3$.

As another approximation, we can try assuming the wave function is constant over the volume of the nucleus. This gives:

$$Prob(r \le b) = \frac{4}{3}\pi b^3 |\psi(0)|^2 \quad (4)$$

$$= \frac{4}{3}\frac{\pi b^3}{\pi a^3} \quad (5)$$

$$= \frac{4}{3}\frac{b^3}{a^3} \quad (6)$$

Thus the two approximations give the same result.

Using $b \approx 10^{-15}$ m and $a \approx 0.5 \times 10^{-10}$ m, we get

$$Prob(r \leq b) \approx \frac{32}{3} \times 10^{-15} = 1.07 \times 10^{-14} \tag{7}$$