

## SPIN 3/2

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 4.52.

Continuing our exploration of higher spin states, we can work out the spin matrix  $S_x$  for spin 3/2. We use the raising and lowering operators first:

$$S_{\pm}|sm\rangle = \hbar\sqrt{s(s+1) - m(m\pm 1)}|sm\pm 1\rangle \quad (1)$$

Here,  $|sm\rangle$  is an eigenstate of  $S^2$  with eigenvalue  $s(s+1)$  and of  $S_z$  with eigenvalue  $m$ . We can work out the effects of  $S_{\pm}$  on the various eigenstates of  $S_z$  for  $s = 3/2$  and get

$$S_+ \left| \frac{3}{2} \frac{3}{2} \right\rangle = 0 \quad (2)$$

$$S_+ \left| \frac{3}{2} \frac{1}{2} \right\rangle = \sqrt{3}\hbar \left| \frac{3}{2} \frac{3}{2} \right\rangle \quad (3)$$

$$S_+ \left| \frac{3}{2} -\frac{1}{2} \right\rangle = 2\hbar \left| \frac{3}{2} \frac{1}{2} \right\rangle \quad (4)$$

$$S_+ \left| \frac{3}{2} -\frac{3}{2} \right\rangle = \sqrt{3}\hbar \left| \frac{3}{2} -\frac{1}{2} \right\rangle \quad (5)$$

$$S_- \left| \frac{3}{2} \frac{3}{2} \right\rangle = \sqrt{3}\hbar \left| \frac{3}{2} \frac{1}{2} \right\rangle \quad (6)$$

$$S_- \left| \frac{3}{2} \frac{1}{2} \right\rangle = 2\hbar \left| \frac{3}{2} -\frac{1}{2} \right\rangle \quad (7)$$

$$S_- \left| \frac{3}{2} -\frac{1}{2} \right\rangle = \sqrt{3}\hbar \left| \frac{3}{2} -\frac{3}{2} \right\rangle \quad (8)$$

$$S_- \left| \frac{3}{2} -\frac{3}{2} \right\rangle = 0 \quad (9)$$

Combining these conditions, we get the matrix forms for  $S_{\pm}$ :

$$S_+ = \hbar \begin{pmatrix} 0 & \sqrt{3} & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & \sqrt{3} \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (10)$$

$$S_- = \hbar \begin{pmatrix} 0 & 0 & 0 & 0 \\ \sqrt{3} & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix} \quad (11)$$

so, since  $S_x = (S_+ + S_-)/2$

$$S_x = \frac{\hbar}{2} \begin{pmatrix} 0 & \sqrt{3} & 0 & 0 \\ \sqrt{3} & 0 & 2 & 0 \\ 0 & 2 & 0 & \sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix} \quad (12)$$

The characteristic equation for the matrix part of  $S_x$  is

$$-\lambda(-\lambda(\lambda^2 - 3) - 2(-2\lambda)) - \sqrt{3}(\sqrt{3}(\lambda^2 - 3)) = 0 \quad (13)$$

$$\lambda^4 - 10\lambda^2 + 9 = 0 \quad (14)$$

$$\lambda = \pm 3, \pm 1 \quad (15)$$

Thus the eigenvalues of  $S_x$  are  $\pm 3\hbar/2$  and  $\pm\hbar/2$  as expected.

#### PINGBACKS

Pingback: Spin matrices: general case