SPIN 3/2

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 4.52.

Continuing our exploration of higher spin states, we can work out the spin matrix S_x for spin 3/2. We use the raising and lowering operators first:

$$S_{\pm}|s\,m\rangle = \hbar\sqrt{s(s+1)-m(m\pm1)}|s\,m\pm1\rangle \tag{1}$$

Here, $|s\,m\rangle$ is an eigenstate of S^2 with eigenvalue $s(s+1)$ and of S_z with eigenvalue m. We can work out the effects of $S_{\pm}$ on the various eigenstates of S_z for $s=3/2$ and get

$$S_+\left|\frac{3}{2}\,\frac{3}{2}\right\rangle = 0 \tag{2}$$
$$S_+\left|\frac{3}{2}\,\frac{1}{2}\right\rangle = \sqrt{3}\hbar\left|\frac{3}{2}\,\frac{3}{2}\right\rangle \tag{3}$$
$$S_+\left|\frac{3}{2}\,-\frac{1}{2}\right\rangle = 2\hbar\left|\frac{3}{2}\,\frac{1}{2}\right\rangle \tag{4}$$
$$S_+\left|\frac{3}{2}\,-\frac{3}{2}\right\rangle = \sqrt{3}\hbar\left|\frac{3}{2}\,-\frac{1}{2}\right\rangle \tag{5}$$
$$S_-\left|\frac{3}{2}\,\frac{3}{2}\right\rangle = \sqrt{3}\hbar\left|\frac{3}{2}\,\frac{1}{2}\right\rangle \tag{6}$$
$$S_-\left|\frac{3}{2}\,\frac{1}{2}\right\rangle = 2\hbar\left|\frac{3}{2}\,-\frac{1}{2}\right\rangle \tag{7}$$
$$S_-\left|\frac{3}{2}\,-\frac{1}{2}\right\rangle = \sqrt{3}\hbar\left|\frac{3}{2}\,-\frac{3}{2}\right\rangle \tag{8}$$
$$S_-\left|\frac{3}{2}\,-\frac{3}{2}\right\rangle = 0 \tag{9}$$

Combining these conditions, we get the matrix forms for $S_{\pm}$:

$$S_+ = \hbar\left(\begin{array}{cccc} 0 & \sqrt{3} & 0 & 0\\ 0 & 0 & 2 & 0\\ 0 & 0 & 0 & \sqrt{3}\\ 0 & 0 & 0 & 0 \end{array}\right) \tag{10}$$

$$S_{-}=\hbar\left(\begin{array}{cccc}0 & 0 & 0 & 0\\ \sqrt{3} & 0 & 0 & 0\\ 0 & 2 & 0 & 0\\ 0 & 0 & \sqrt{3} & 0\end{array}\right) \tag{11}$$

so, since $S_x=(S_{+}+S_{-})/2$

$$S_x=\frac{\hbar}{2}\left(\begin{array}{cccc}0 & \sqrt{3} & 0 & 0\\ \sqrt{3} & 0 & 2 & 0\\ 0 & 2 & 0 & \sqrt{3}\\ 0 & 0 & \sqrt{3} & 0\end{array}\right) \tag{12}$$

The characteristic equation for the matrix part of S_x is

$$-\lambda(-\lambda(\lambda^2-3)-2(-2\lambda))-\sqrt{3}(\sqrt{3}(\lambda^2-3))=0 \tag{13}$$

$$\lambda^4-10\lambda^2+9=0 \tag{14}$$

$$\lambda=\pm3,\ \pm1 \tag{15}$$

Thus the eigenvalues of S_x are $\pm3\hbar/2$ and $\pm\hbar/2$ as expected.

PINGBACKS

Pingback: Spin matrices: general case