SPIN 3/2

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 4.52.

Continuing our exploration of higher spin states, we can work out the spin matrix S_x for spin 3/2. We use the raising and lowering operators first:

$$S_{\pm}|s\,m\rangle = \hbar\sqrt{s(s+1)-m(m\pm1)}|s\,m\pm1\rangle \tag{1}$$

Here, $|s\,m\rangle$ is an eigenstate of S^2 with eigenvalue $s(s+1)$ and of S_z with eigenvalue m. We can work out the effects of $S_{\pm}$ on the various eigenstates of S_z for $s=3/2$ and get

$$S_+\left|\frac{3}{2}\,\frac{3}{2}\right\rangle = 0 \tag{2}$$

$$S_+\left|\frac{3}{2}\,\frac{1}{2}\right\rangle = \sqrt{3}\hbar\left|\frac{3}{2}\,\frac{3}{2}\right\rangle \tag{3}$$

$$S_+\left|\frac{3}{2}\,-\frac{1}{2}\right\rangle = 2\hbar\left|\frac{3}{2}\,\frac{1}{2}\right\rangle \tag{4}$$

$$S_+\left|\frac{3}{2}\,-\frac{3}{2}\right\rangle = \sqrt{3}\hbar\left|\frac{3}{2}\,-\frac{1}{2}\right\rangle \tag{5}$$

$$S_-\left|\frac{3}{2}\,\frac{3}{2}\right\rangle = \sqrt{3}\hbar\left|\frac{3}{2}\,\frac{1}{2}\right\rangle \tag{6}$$

$$S_-\left|\frac{3}{2}\,\frac{1}{2}\right\rangle = 2\hbar\left|\frac{3}{2}\,-\frac{1}{2}\right\rangle \tag{7}$$

$$S_-\left|\frac{3}{2}\,-\frac{1}{2}\right\rangle = \sqrt{3}\hbar\left|\frac{3}{2}\,-\frac{3}{2}\right\rangle \tag{8}$$

$$S_-\left|\frac{3}{2}\,-\frac{3}{2}\right\rangle = 0 \tag{9}$$

Combining these conditions, we get the matrix forms for $S_{\pm}$:

$$S_+ = \hbar\begin{pmatrix} 0 & \sqrt{3} & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & \sqrt{3} \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{10}$$

$$S_- = \hbar \begin{pmatrix} 0 & 0 & 0 & 0 \\ \sqrt{3} & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix} \tag{11}$$

so, since $S_x = (S_+ + S_-)/2$

$$S_x = \frac{\hbar}{2} \begin{pmatrix} 0 & \sqrt{3} & 0 & 0 \\ \sqrt{3} & 0 & 2 & 0 \\ 0 & 2 & 0 & \sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix} \tag{12}$$

The characteristic equation for the matrix part of S_x is

$$-\lambda(-\lambda(\lambda^2 - 3) - 2(-2\lambda)) - \sqrt{3}(\sqrt{3}(\lambda^2 - 3)) = 0 \tag{13}$$

$$\lambda^4 - 10\lambda^2 + 9 = 0 \tag{14}$$

$$\lambda = \pm 3,\ \pm 1 \tag{15}$$

Thus the eigenvalues of S_x are $\pm 3\hbar/2$ and $\pm\hbar/2$ as expected.

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