

IDENTICAL PARTICLES: FERMIONS AND BOSONS

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 5.4.

One of the more unusual postulates of quantum mechanics is that particles of the same type (for example, 2 electrons) are actually identical, in the sense that we cannot distinguish one from the other by any means at all. This is a radical departure from classical mechanics, where all particles are, in principle, different, even if they have the same physical properties. In classical mechanics, it makes sense to talk about 'electron #1' and 'electron #2', but in quantum mechanics, all electrons (or particles of other types such as protons and neutrons) are exactly the same, and it is impossible to label them in any way.

Up to now, we've been considering mainly one-particle systems moving in potentials, so the issue of identifying one of a group of particles hasn't arisen. We've essentially assumed that the one particle (usually an electron) is the only particle in the universe, so we don't need to think about labelling it.

For two (or more) particles, though, we do need to think about how to describe the quantum state in such a way that, even from the mathematics, we can't tell which particle is which. For a single particle moving in a potential, we can solve the time-independent Schrödinger equation and get a set of stationary states. Once we've done that, we can generate the time-dependent solution in the usual manner by constructing a series:

$$\Psi(\mathbf{r}, t) = \sum_{n=1}^{\infty} c_n \psi_n(\mathbf{r}) e^{-iE_n t/\hbar} \quad (1)$$

If we have 2 particles, we need to introduce two spatial coordinates, one for each particle. Now suppose that each particle is in one of the stationary states, say ψ_1 and ψ_2 . If the particles are really indistinguishable, then it shouldn't be possible to tell which particle is in which state. That is, trying a total wave function of the form $\psi(\mathbf{r}_a, \mathbf{r}_b) = \psi_1(\mathbf{r}_a)\psi_2(\mathbf{r}_b)$ won't work, because we've associated particle a with state ψ_1 and b with state ψ_2 .

One way of mixing things up is to take a symmetric or anti-symmetric combination of the two states. That is

$$\psi_{\pm}(\mathbf{r}_a, \mathbf{r}_b) = A[\psi_1(\mathbf{r}_a)\psi_2(\mathbf{r}_b) \pm \psi_2(\mathbf{r}_a)\psi_1(\mathbf{r}_b)] \quad (2)$$

where A is a normalization constant. Assuming the individual ψ functions are orthonormal, we can work out A . Using the orthonormal properties of the $\psi_{1,2}$ functions, we get, assuming $\psi_1 \neq \psi_2$:

$$\int \psi_{\pm}^* \psi_{\pm} d^3\mathbf{r}_a d^3\mathbf{r}_b = |A|^2 \int [\psi_1(\mathbf{r}_a)\psi_2(\mathbf{r}_b) \pm \psi_2(\mathbf{r}_a)\psi_1(\mathbf{r}_b)]^* \times \quad (3)$$

$$[\psi_1(\mathbf{r}_a)\psi_2(\mathbf{r}_b) \pm \psi_2(\mathbf{r}_a)\psi_1(\mathbf{r}_b)] d^3\mathbf{r}_a d^3\mathbf{r}_b \quad (4)$$

$$= 2|A|^2 \quad (5)$$

$$= 1 \quad (6)$$

so $A = 1/\sqrt{2}$.

In the case where $\psi_1 = \psi_2 = \psi$, the minus sign results in the total wave function being zero, so we have only the case with the plus sign. In this case, we get

$$\int \psi_{\pm}^* \psi_{\pm} d^3\mathbf{r}_1 d^3\mathbf{r}_2 = |A|^2 \int [2\psi(\mathbf{r}_a)\psi(\mathbf{r}_b)]^* [2\psi(\mathbf{r}_a)\psi(\mathbf{r}_b)] d^3\mathbf{r}_a d^3\mathbf{r}_b \quad (7)$$

$$= 4|A|^2 = 1 \quad (8)$$

so here $A = 1/2$.

Particles for which we take the plus sign above are known as *bosons*, and particles using the minus sign are known as *fermions*.

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