INFINITE SQUARE WELL: 2 PARTICLE SYSTEMS

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 5.5.

As an example of the differences between distinguishable particles and identical particles, consider 2 particles in the infinite square well. The wave functions for a single particle are

$$\psi(x)=\sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a} \tag{0.1}$$

where a is the width of the well. The energy (eigenvalue) of this state is

$$E_n=\frac{n^2\pi^2\hbar^2}{2ma^2}\equiv n^2K \tag{0.2}$$

If we have two noninteracting particles, the hamiltonian consists of terms for each particle:

$$H\Psi=-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x_1^2}-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x_2^2} \tag{0.3}$$

for $0<x_{1,2}<a$.

If the particles are distinguishable, then there is no need to arrange the wave functions so that we can't tell which particle is in which state. That is, we can form a simple product:

$$\psi_d(x_1,x_2)=\psi_{n_1}(x_1)\,\psi_{n_2}(x_2) \tag{0.4}$$

If we apply the hamiltonian to this function, the total energy is just the sum of the two individual energies:

$$H\psi_d(x_1,x_2)=E_{n_1}+E_{n_2} \tag{0.5}$$

The ground state has $n_1=n_2=1$, and thus an energy of $2K$. The first excited state has either $n_1=1$; $n_2=2$ or $n_1=2$; $n_2=1$. Since the particles are not identical, these are distinct states, so the first excited state has a degeneracy of 2, and an energy of $5K$. After that, the next state has $n_1=$

$n_2 = 2$ (degeneracy 1, energy $8K$), then $n_1 = 1$; $n_2 = 3$ or $n_1 = 3$; $n_2 = 1$ (degeneracy 2, energy $10K$).

If the particles are bosons, then they are identical and the wave function is a symmetric sum, so we get

$$\psi_b(x_1,x_2) = \frac{1}{\sqrt{2}}\left[\psi_{n_1}(x_1)\,\psi_{n_2}(x_2) + \psi_{n_1}(x_2)\,\psi_{n_2}(x_1)\right] \tag{0.6}$$

The ground state has $n_1 = n_2 = 1$, and thus an energy of $2K$ as before. The first excited state has $n_1 = 1$; $n_2 = 2$. This time, however, setting $n_1 = 2$; $n_2 = 1$ does not give us a separate state, as you can see by plugging in the numbers into ψ_b. Thus the $5K$ energy state has degeneracy 1, not 2.

We then get $n_1 = n_2 = 2$ (degeneracy 1, energy $8K$), then $n_1 = 1$; $n_2 = 3$ (degeneracy 1, energy $10K$).

For fermions, the wave function is an antisymmetric sum:

$$\psi_f(x_1,x_2) = \frac{1}{\sqrt{2}}\left[\psi_{n_1}(x_1)\,\psi_{n_2}(x_2) - \psi_{n_1}(x_2)\,\psi_{n_2}(x_1)\right] \tag{0.7}$$

This time, whenever $n_1 = n_2$ the wave function $\psi_f = 0$, so for fermions it is impossible to have two particles in the same state. This is a quite general result and is known as the *Pauli exclusion principle*. For the infinite square well, the ground state for fermions is therefore $n_1 = 1$; $n_2 = 2$, with energy $5K$ and degeneracy 1. In this case, using $n_1 = 2$; $n_2 = 1$ does give a different wave function, but it is simply the negative of the original, so differs from the original only by a phase factor which disappears when taking the square modulus.

The next state occurs at $n_1 = 1$; $n_2 = 3$ (degeneracy 1, energy $10K$), then $n_1 = 2$; $n_2 = 3$ (degeneracy 1, energy $13K$).

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