INFINITE SQUARE WELL: 2 PARTICLE SYSTEMS

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 5.5.

As an example of the differences between distinguishable particles and identical particles, consider 2 particles in the infinite square well. The wave functions for a single particle are

$$\psi(x) = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a} \tag{1}$$

where a is the width of the well. The energy (eigenvalue) of this state is

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2ma^2} \equiv n^2 K \tag{2}$$

If we have two noninteracting particles, the hamiltonian consists of terms for each particle:

$$H\Psi = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x_1^2} - \frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x_2^2} \tag{3}$$

for $0 < x_{1,2} < a$.

If the particles are distinguishable, then there is no need to arrange the wave functions so that we can't tell which particle is in which state. That is, we can form a simple product:

$$\psi_d(x_1, x_2) = \psi_{n_1}(x_1) \psi_{n_2}(x_2) \tag{4}$$

If we apply the hamiltonian to this function, the total energy is just the sum of the two individual energies:

$$H\psi_d(x_1, x_2) = E_{n_1} + E_{n_2} \tag{5}$$

The ground state has $n_1 = n_2 = 1$, and thus an energy of $2K$. The first excited state has either $n_1 = 1$; $n_2 = 2$ or $n_1 = 2$; $n_2 = 1$. Since the particles are not identical, these are distinct states, so the first excited state has a degeneracy of 2, and an energy of $5K$. After that, the next state has $n_1 = n_2 = 2$ (degeneracy 1, energy $8K$), then $n_1 = 1$; $n_2 = 3$ or $n_1 = 3$; $n_2 = 1$ (degeneracy 2, energy $10K$).

If the particles are bosons, then they are identical and the wave function is a symmetric sum, so we get

$$\psi_b(x_1, x_2) = \frac{1}{\sqrt{2}} [\psi_{n_1}(x_1)\psi_{n_2}(x_2) + \psi_{n_1}(x_2)\psi_{n_2}(x_1)] \qquad (6)$$

The ground state has $n_1 = n_2 = 1$, and thus an energy of $2K$ as before. The first excited state has $n_1 = 1$; $n_2 = 2$. This time, however, setting $n_1 = 2$; $n_2 = 1$ does not give us a separate state, as you can see by plugging in the numbers into ψ_b. Thus the $5K$ energy state has degeneracy 1, not 2.

We then get $n_1 = n_2 = 2$ (degeneracy 1, energy $8K$), then $n_1 = 1$; $n_2 = 3$ (degeneracy 1, energy $10K$).

For fermions, the wave function is an antisymmetric sum:

$$\psi_f(x_1, x_2) = \frac{1}{\sqrt{2}} [\psi_{n_1}(x_1)\psi_{n_2}(x_2) - \psi_{n_1}(x_2)\psi_{n_2}(x_1)] \qquad (7)$$

This time, whenever $n_1 = n_2$ the wave function $\psi_f = 0$, so for fermions it is impossible to have two particles in the same state. This is a quite general result and is known as the *Pauli exclusion principle*. For the infinite square well, the ground state for fermions is therefore $n_1 = 1$; $n_2 = 2$, with energy $5K$ and degeneracy 1. In this case, using $n_1 = 2$; $n_2 = 1$ does give a different wave function, but it is simply the negative of the original, so differs from the original only by a phase factor which disappears when taking the square modulus.

The next state occurs at $n_1 = 1$; $n_2 = 3$ (degeneracy 1, energy $10K$), then $n_1 = 2$; $n_2 = 3$ (degeneracy 1, energy $13K$).

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