

## INFINITE SQUARE WELL: 2 PARTICLE SYSTEMS

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 5.5.

As an example of the differences between distinguishable particles and identical particles, consider 2 particles in the infinite square well. The wave functions for a single particle are

$$\psi(x) = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a} \quad (1)$$

where  $a$  is the width of the well. The energy (eigenvalue) of this state is

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2ma^2} \equiv n^2 K \quad (2)$$

If we have two noninteracting particles, the hamiltonian consists of terms for each particle:

$$H\Psi = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x_1^2} - \frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x_2^2} \quad (3)$$

for  $0 < x_{1,2} < a$ .

If the particles are distinguishable, then there is no need to arrange the wave functions so that we can't tell which particle is in which state. That is, we can form a simple product:

$$\Psi_d(x_1, x_2) = \Psi_{n_1}(x_1) \Psi_{n_2}(x_2) \quad (4)$$

If we apply the hamiltonian to this function, the total energy is just the sum of the two individual energies:

$$H\Psi_d(x_1, x_2) = E_{n_1} + E_{n_2} \quad (5)$$

The ground state has  $n_1 = n_2 = 1$ , and thus an energy of  $2K$ . The first excited state has either  $n_1 = 1; n_2 = 2$  or  $n_1 = 2; n_2 = 1$ . Since the particles are not identical, these are distinct states, so the first excited state has a degeneracy of 2, and an energy of  $5K$ . After that, the next state has  $n_1 = n_2 = 2$  (degeneracy 1, energy  $8K$ ), then  $n_1 = 1; n_2 = 3$  or  $n_1 = 3; n_2 = 1$  (degeneracy 2, energy  $10K$ ).

If the particles are bosons, then they are identical and the wave function is a symmetric sum, so we get

$$\psi_b(x_1, x_2) = \frac{1}{\sqrt{2}} [\psi_{n_1}(x_1) \psi_{n_2}(x_2) + \psi_{n_1}(x_2) \psi_{n_2}(x_1)] \quad (6)$$

The ground state has  $n_1 = n_2 = 1$ , and thus an energy of  $2K$  as before. The first excited state has  $n_1 = 1; n_2 = 2$ . This time, however, setting  $n_1 = 2; n_2 = 1$  does not give us a separate state, as you can see by plugging in the numbers into  $\psi_b$ . Thus the  $5K$  energy state has degeneracy 1, not 2.

We then get  $n_1 = n_2 = 2$  (degeneracy 1, energy  $8K$ ), then  $n_1 = 1; n_2 = 3$  (degeneracy 1, energy  $10K$ ).

For fermions, the wave function is an antisymmetric sum:

$$\psi_f(x_1, x_2) = \frac{1}{\sqrt{2}} [\psi_{n_1}(x_1) \psi_{n_2}(x_2) - \psi_{n_1}(x_2) \psi_{n_2}(x_1)] \quad (7)$$

This time, whenever  $n_1 = n_2$  the wave function  $\psi_f = 0$ , so for fermions it is impossible to have two particles in the same state. This is a quite general result and is known as the *Pauli exclusion principle*. For the infinite square well, the ground state for fermions is therefore  $n_1 = 1; n_2 = 2$ , with energy  $5K$  and degeneracy 1. In this case, using  $n_1 = 2; n_2 = 1$  does give a different wave function, but it is simply the negative of the original, so differs from the original only by a phase factor which disappears when taking the square modulus.

The next state occurs at  $n_1 = 1; n_2 = 3$  (degeneracy 1, energy  $10K$ ), then  $n_1 = 2; n_2 = 3$  (degeneracy 1, energy  $13K$ ).

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