FERMIONS AND BOSONS: N-PARTICLE SYSTEMS

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Section 5.1.2 & Problem 5.7.

The extension of the two-particle wave functions for identical particles to higher numbers of particles is relatively straightforward. The condition which must be satisfied is that, for bosons, the interchange of any two particles (that is, the interchange of their spatial coordinates) gives the same wave function, while for fermions, the interchange of any two particles gives the *negative* of the original wave function.

The boson case can be realized by generating the $n!$ permutations of n objects and including a term for each permutation. For 3 particles, there are $3! = 6$ permutations, so the wave function is

$$\begin{aligned}\sqrt{6}\psi_b(x_a,x_b,x_c) = \psi_{1a}\psi_{2b}\psi_{3c} + \psi_{1b}\psi_{2c}\psi_{3a} + \psi_{1c}\psi_{2a}\psi_{3b} + \\ \psi_{1a}\psi_{2c}\psi_{3b} + \psi_{1c}\psi_{2b}\psi_{3a} + \psi_{1b}\psi_{2a}\psi_{3c}\end{aligned} \tag{0.1}$$

where ψ_{1a} is the wave function for a particle at location x_a in state ψ_1 and so on. The $\sqrt{6}$ is for normalization; for n particles, this factor is $\sqrt{n!}$.

For fermions, all even permutations (resulting from an even number of swaps) are positive and all odd permutations are negative. In the example above, the last three terms result from a single swap, while the first three result from two swaps, so we can generate the fermion wave function by making the last 3 terms negative:

$$\begin{aligned}\sqrt{6}\psi_f(x_a,x_b,x_c) = \psi_{1a}\psi_{2b}\psi_{3c} + \psi_{1b}\psi_{2c}\psi_{3a} + \psi_{1c}\psi_{2a}\psi_{3b} - \\ \psi_{1a}\psi_{2c}\psi_{3b} - \psi_{1c}\psi_{2b}\psi_{3a} - \psi_{1b}\psi_{2a}\psi_{3c}\end{aligned} \tag{0.2}$$

The fermion case can also be generated by a *Slater determinant* which, for the 3-particle case is

$$\psi_f = \frac{1}{\sqrt{6}}\begin{vmatrix} \psi_{1a} & \psi_{1b} & \psi_{1c} \\ \psi_{2a} & \psi_{2b} & \psi_{2c} \\ \psi_{3a} & \psi_{3b} & \psi_{3c} \end{vmatrix} \tag{0.3}$$

This works for any number of particles. The general rule is: mth row of the determinant consists of ψ_m evaluated at each coordinate in turn.

For distinguishable, non-interacting particles, the wave function is a simple product:

$$\psi_d = \psi_{1a}\psi_{2b}\psi_{3c} \tag{0.4}$$

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