BULK MODULUS IN THE ELECTRON GAS MODEL

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 5.17.

The *bulk modulus* of a substance measures its resistance to compression. It's defined by

$$B=-V\frac{dP}{dV} \tag{0.1}$$

Since increasing the pressure causes a decrease in volume, the derivative is negative, so B is positive. The extra factor of V means that the bulk modulus measures the amount of pressure change required for a relative decrease in volume (that is, the fraction of the original volume). Thus a large bulk modulus means that a larger pressure change is required to compress the substance by the same relative amount, making it more resistant to compression.

For an electron gas, we have the relation between P and V:

$$P=\frac{\hbar^{2}\left(3\pi^{2}\right)^{2/3}}{5m}\left(\frac{Nq}{V}\right)^{5/3} \tag{0.2}$$

By taking the derivative we get

$$B = -V\left(-\frac{5}{3}\right)\frac{\hbar^{2}\left(3\pi^{2}\right)^{2/3}}{5m}(Nq)^{5/3}V^{-8/3} \tag{0.3}$$

$$= \frac{5}{3}P \tag{0.4}$$

For copper, the degeneracy pressure is $P=3.84\times10^{10}\text{N m}^{-2}$ so the bulk modulus is $B=6.39\times10^{10}\text{ N m}^{-2}$. The experimentally measured value is $13.4\times10^{10}\text{ N m}^{-2}$. The agreement is within roughly a factor of 2 which isn't bad considering how crude the electron gas model is.

Incidentally, the bulk modulus of air is about 10^{5} N m^{-2} and that of water is around $2.2\times10^{9}\text{ N m}^{-2}$. It's sometimes said that water is an incompressible fluid, which isn't quite true, but it's still only about 50 times as compressible as a solid metal such as copper. By comparison, air is around one million times as compressible as copper.