

BULK MODULUS IN THE ELECTRON GAS MODEL

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 5.17.

The *bulk modulus* of a substance measures its resistance to compression. It's defined by

$$(1) \quad B = -V \frac{dP}{dV}$$

Since increasing the pressure causes a decrease in volume, the derivative is negative, so B is positive. The extra factor of V means that the bulk modulus measures the amount of pressure change required for a relative decrease in volume (that is, the fraction of the original volume). Thus a large bulk modulus means that a larger pressure change is required to compress the substance by the same relative amount, making it more resistant to compression.

For an electron gas, we have the relation between P and V :

$$(2) \quad P = \frac{\hbar^2 (3\pi^2)^{2/3}}{5m} \left(\frac{Nq}{V} \right)^{5/3}$$

By taking the derivative we get

$$(3) \quad B = -V \left(-\frac{5}{3} \right) \frac{\hbar^2 (3\pi^2)^{2/3}}{5m} (Nq)^{5/3} V^{-8/3}$$

$$(4) \quad = \frac{5}{3} P$$

For copper, the degeneracy pressure is $P = 3.84 \times 10^{10} \text{ N m}^{-2}$ so the bulk modulus is $B = 6.39 \times 10^{10} \text{ N m}^{-2}$. The experimentally measured value is $13.4 \times 10^{10} \text{ N m}^{-2}$. The agreement is within roughly a factor of 2 which isn't bad considering how crude the electron gas model is.

Incidentally, the bulk modulus of air is about 10^5 N m^{-2} and that of water is around $2.2 \times 10^9 \text{ N m}^{-2}$. It's sometimes said that water is an incompressible fluid, which isn't quite true, but it's still only about 50 times as

compressible as a solid metal such as copper. By comparison, air is around one million times as compressible as copper.