STEFAN-BOLTZMANN LAW

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 5.31.

We worked out the energy density in blackbody radiation as a function of frequency. The energy density is (number of photons with frequency ω) (energy of 1 photon with frequ
which is

$$\frac{n_\omega \hbar\omega}{V} = \rho(\omega)\, d\omega \tag{1}$$

$$= \frac{1}{\left(e^{\hbar\omega/k_BT}-1\right)} \frac{\hbar\omega^3}{\pi^2 c^3} d\omega \tag{2}$$

To find the total energy density we integrate over the frequency:

$$\frac{E}{V} = \frac{\hbar}{\pi^2 c^3} \int_0^\infty \frac{\omega^3}{\left(e^{\hbar\omega/k_BT}-1\right)} d\omega \tag{3}$$

This integral can be done by changing variables to $x = \hbar\omega/k_BT$ and then using the formula 5.110 in Griffiths giving the result in terms of the Riemann zeta function. However, plugging the integral directly into Maple gives the result immediately:

$$\frac{E}{V} = \frac{\pi^2 k_B^4}{15\hbar^3 c^3} T^4 \tag{4}$$

This is the Stefan-Boltzmann law which says that the energy density increases as the fourth power of the temperature.

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