

STEFAN-BOLTZMANN LAW

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 5.31.

We worked out the energy density in blackbody radiation as a function of frequency. The energy density is (number of photons with frequency ω) (energy of 1 photon with frequency ω) which is

$$\begin{aligned} (1) \quad \frac{n_\omega \hbar \omega}{V} &= \rho(\omega) d\omega \\ (2) \quad &= \frac{1}{(e^{\hbar\omega/k_B T} - 1)} \frac{\hbar \omega^3}{\pi^2 c^3} d\omega \end{aligned}$$

To find the total energy density we integrate over the frequency:

$$(3) \quad \frac{E}{V} = \frac{\hbar}{\pi^2 c^3} \int_0^\infty \frac{\omega^3}{(e^{\hbar\omega/k_B T} - 1)} d\omega$$

This integral can be done by changing variables to $x = \hbar\omega/k_B T$ and then using the formula 5.110 in Griffiths giving the result in terms of the Riemann zeta function. However, plugging the integral directly into Maple gives the result immediately:

$$(4) \quad \frac{E}{V} = \frac{\pi^2 k_B^4}{15 \hbar^3 c^3} T^4$$

This is the Stefan-Boltzmann law which says that the energy density increases as the fourth power of the temperature.

PINGBACKS

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