EXCHANGE FORCE: HARMONIC OSCILLATOR

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 5.32.

Here's another example of calculating mean separation distances for two particles, this time in the harmonic oscillator potential. We have one particle in the ground state and the other in the first excited state.

We've already worked out the relevant one-particle averages, so we'll quote them here. In the ground state

$$\langle x^2\rangle_0 = \frac{\hbar}{2m\omega} \tag{1}$$

$$\langle x\rangle_0 = 0 \tag{2}$$

In the first excited state

$$\langle x^2\rangle_1 = \frac{3\hbar}{2m\omega} \tag{3}$$

$$\langle x\rangle_1 = 0 \tag{4}$$

For distinguishable particles,

$$\left\langle (x_a - x_b)^2 \right\rangle = \left\langle x^2 \right\rangle_0 + \left\langle x^2 \right\rangle_1 - 2\left\langle x \right\rangle_0 \left\langle x \right\rangle_1 \tag{5}$$

$$= \frac{2\hbar}{m\omega} \tag{6}$$

For fermions and bosons, we need the cross-term average.

$$\langle x\rangle_{01} = \langle\,\psi_0|\,x\,|\psi_1\,\rangle \tag{7}$$

$$= \sqrt{\frac{2}{\pi}\frac{m\omega}{\hbar}}\int_{-\infty}^{\infty} x^2 e^{-m\omega x^2/\hbar} dx \tag{8}$$

$$= \sqrt{\frac{\hbar}{2m\omega}} \tag{9}$$

For bosons,

$$\left\langle (x_a - x_b)^2 \right\rangle = \left\langle x^2 \right\rangle_0 + \left\langle x^2 \right\rangle_1 - 2\left\langle x \right\rangle_0 \left\langle x \right\rangle_1 - 2\left\langle x \right\rangle_{01}^2 \tag{10}$$

$$= \frac{\hbar}{m\omega} \tag{11}$$

For fermions

$$\left\langle (x_a - x_b)^2 \right\rangle = \left\langle x^2 \right\rangle_0 + \left\langle x^2 \right\rangle_1 - 2\left\langle x \right\rangle_0 \left\langle x \right\rangle_1 + 2\left\langle x \right\rangle_{01}^2 \tag{12}$$

$$= \frac{3\hbar}{m\omega} \tag{13}$$