ELECTRON GAS IN A 2-D INFINITE SQUARE WELL

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 5.34.

By analogy with the ideal gas in the 3-d infinite square well, we can work out the Fermi energy for a 2-d infinite square well. In this case, k-space occupies the first quadrant of the plane, and each cell in k-space occupies an area of $\pi^2/l_x l_y = \pi^2/A$, where A is the area of the square well and l_x and l_y are its dimensions.

If the space is filled with electrons in the ground state, they will fill the first quadrant of a circle with radius k_F. As before, the number of states required if we have N atoms with q electrons per atom is $Nq/2$ (divided by 2 because of the spin degeneracy) so the total area required is

$$\frac{Nq}{2}\frac{\pi^2}{A} = \frac{\pi k_F^2}{4} \tag{1}$$

$$k_F = \sqrt{2\sigma\pi} \tag{2}$$

$$\sigma \equiv \frac{Nq}{A} \tag{3}$$

The Fermi energy is then

$$E_F = \frac{\hbar^2 k_F^2}{2m} \tag{4}$$

$$= \frac{\hbar^2 \sigma \pi}{m} \tag{5}$$