

## HARMONIC OSCILLATOR: FIRST ORDER PERTURBATION

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 6.2.

This is a simple example of applying first order perturbation theory to the harmonic oscillator. The energy levels of an unperturbed oscillator are

$$(1) \quad E_{n0} = \left(n + \frac{1}{2}\right) \hbar\omega$$

where  $\omega = \sqrt{k/m}$  and the potential is  $V = \frac{1}{2}kx^2$ . If we perturb the potential by changing  $k$  slightly, so the new potential is

$$(2) \quad V' = \frac{1}{2}(1 + \varepsilon)kx^2$$

then, of course, it's easy to find the exact energy levels just by changing  $k$  in the original formula:

$$(3) \quad E_n = \left(n + \frac{1}{2}\right) \hbar\omega'$$

$$(4) \quad = \left(n + \frac{1}{2}\right) \hbar\sqrt{\frac{k(1 + \varepsilon)}{m}}$$

We can expand the square root in a power series:

$$(5) \quad E_n = \left(n + \frac{1}{2}\right) \hbar\sqrt{\frac{k}{m}} \left[1 + \frac{\varepsilon}{2} - \frac{\varepsilon^2}{8} + \dots\right]$$

From first order perturbation theory, the change to the energy is (since the perturbation in the potential here is  $V' - V = \frac{1}{2}\varepsilon kx^2 = \varepsilon V$ ):

$$(6) \quad E_{n1} = \langle n0 | \varepsilon V | n0 \rangle$$

We could do the integral implied here, but we've already worked out the mean value of the potential for the harmonic oscillator using the virial theorem, and we know that  $\langle V \rangle = \langle T \rangle = E_n/2$  so

$$(7) \quad E_{n1} = \frac{\varepsilon}{2} E_{n0}$$

$$(8) \quad = \frac{\varepsilon}{2} \left( n + \frac{1}{2} \right) \hbar \sqrt{\frac{k}{m}}$$

This is the first order term in  $\varepsilon$  in the series expansion above.

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