HARMONIC OSCILLATOR: FIRST ORDER PERTURBATION

Link to: [physicspages home page](#).
To leave a comment or report an error, please use the auxiliary blog.
Post date: 26 Jul 2013.

This is a simple example of applying first order perturbation theory to the harmonic oscillator. The energy levels of an unperturbed oscillator are

\[ E_{n0} = \left( n + \frac{1}{2} \right) \hbar \omega \]  

where \( \omega = \sqrt{\frac{k}{m}} \) and the potential is \( V = \frac{1}{2} k x^2 \). If we perturb the potential by changing \( k \) slightly, so the new potential is

\[ V' = \frac{1}{2} (1 + \epsilon) k x^2 \]  

then, of course, it’s easy to find the exact energy levels just by changing \( k \) in the original formula:

\[ E_n = \left( n + \frac{1}{2} \right) \hbar \omega' = \left( n + \frac{1}{2} \right) \hbar \sqrt{\frac{k (1 + \epsilon)}{m}} \]  

We can expand the square root in a power series:

\[ E_n = \left( n + \frac{1}{2} \right) \hbar \sqrt{\frac{k}{m}} \left[ 1 + \frac{\epsilon}{2} - \frac{\epsilon^2}{8} + \ldots \right] \]  

From first order perturbation theory, the change to the energy is (since the perturbation in the potential here is \( V' - V = \frac{1}{2} \epsilon k x^2 = \epsilon V \)):

\[ E_{n1} = \langle n0 | \epsilon V | n0 \rangle \]  

We could do the integral implied here, but we’ve already worked out the mean value of the potential for the harmonic oscillator using the virial theorem, and we know that \( \langle V \rangle = \langle T \rangle = E_n/2 \) so
$E_{n1} = \frac{\epsilon}{2} E_{n0}$ \hspace{1cm} (7)

$= \frac{\epsilon}{2} \left( n + \frac{1}{2} \right) \hbar \sqrt{\frac{k}{m}}$ \hspace{1cm} (8)

This is the first order term in $\epsilon$ in the series expansion above.

PINGBACKS

Pingback: Second order non-degenerate perturbation theory