HARMONIC OSCILLATOR: FIRST ORDER PERTURBATION

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 6.2.

This is a simple example of applying first order perturbation theory to the harmonic oscillator. The energy levels of an unperturbed oscillator are

$$E_{n0} = \left(n + \frac{1}{2} \right) \hbar \omega \tag{1}$$

where $\omega = \sqrt{k/m}$ and the potential is $V = \frac{1}{2} k x^2$. If we perturb the potential by changing k slightly, so the new potential is

$$V' = \frac{1}{2} (1 + \varepsilon) k x^2 \tag{2}$$

then, of course, it's easy to find the exact energy levels just by changing k in the original formula:

$$E_n = \left(n + \frac{1}{2} \right) \hbar \omega' \tag{3}$$

$$= \left(n + \frac{1}{2} \right) \hbar \sqrt{\frac{k(1+\varepsilon)}{m}} \tag{4}$$

We can expand the square root in a power series:

$$E_n = \left(n + \frac{1}{2} \right) \hbar \sqrt{\frac{k}{m}} \left[1 + \frac{\varepsilon}{2} - \frac{\varepsilon^2}{8} + \dots \right] \tag{5}$$

From first order perturbation theory, the change to the energy is (since the perturbation in the potential here is $V' - V = \frac{1}{2} \varepsilon k x^2 = \varepsilon V$):

$$E_{n1} = \langle n0 | \varepsilon V | n0 \rangle \tag{6}$$

We could do the integral implied here, but we've already worked out the mean value of the potential for the harmonic oscillator using the virial theorem, and we know that $\langle V \rangle = \langle T \rangle = E_n / 2$ so

$$E_{n1} = \frac{\varepsilon}{2} E_{n0} \tag{7}$$

$$= \frac{\varepsilon}{2} \left(n + \frac{1}{2} \right) \hbar \sqrt{\frac{k}{m}} \tag{8}$$

This is the first order term in ε in the series expansion above.

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