HARMONIC OSCILLATOR: FIRST ORDER PERTURBATION

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 6.2.

This is a simple example of applying first order perturbation theory to the harmonic oscillator. The energy levels of an unperturbed oscillator are

$$E_{n0}=\left(n+\frac{1}{2}\right)\hbar\omega \tag{0.1}$$

where $\omega=\sqrt{k/m}$ and the potential is $V=\frac{1}{2}kx^2$. If we perturb the potential by changing k slightly, so the new potential is

$$V'=\frac{1}{2}(1+\varepsilon)kx^2 \tag{0.2}$$

then, of course, it's easy to find the exact energy levels just by changing k in the original formula:

$$E_n=\left(n+\frac{1}{2}\right)\hbar\omega' \tag{0.3}$$

$$=\left(n+\frac{1}{2}\right)\hbar\sqrt{\frac{k(1+\varepsilon)}{m}} \tag{0.4}$$

We can expand the square root in a power series:

$$E_n=\left(n+\frac{1}{2}\right)\hbar\sqrt{\frac{k}{m}}\left[1+\frac{\varepsilon}{2}-\frac{\varepsilon^2}{8}+\ldots\right] \tag{0.5}$$

From first order perturbation theory, the change to the energy is (since the perturbation in the potential here is $V'-V=\frac{1}{2}\varepsilon kx^2=\varepsilon V$):

$$E_{n1}=\langle n0|\,\varepsilon V\,|n0\rangle \tag{0.6}$$

We could do the integral implied here, but we've already worked out the mean value of the potential for the harmonic oscillator using the virial theorem, and we know that $\langle V\rangle=\langle T\rangle=E_n/2$ so

$$E_{n1} = \frac{\varepsilon}{2}E_{n0} \tag{0.7}$$

$$= \frac{\varepsilon}{2}\left(n+\frac{1}{2}\right)\hbar\sqrt{\frac{k}{m}} \tag{0.8}$$

This is the first order term in ε in the series expansion above.

PINGBACKS

Pingback: Second order non-degenerate perturbation theory