PERTURBATION THEORY FOR HIGHER-LEVEL DEGENERATE SYSTEMS

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 6.10.

The analysis of degenerate pertrubation theory for a doubly degenerate state generalizes fairly easily to higher levels of degeneracy. The equations we got for the doubly degenerate case are

$$\begin{aligned} \alpha W_{aa}+\beta W_{ab} &= \alpha E_{n1} \quad (1)\\ \alpha W_{ba}+\beta W_{bb} &= \beta E_{n1} \quad (2) \end{aligned}$$

where

$$W_{ij}\equiv\langle i0|V|j0\rangle \quad (3)$$

and V is the perturbation in the potential and E_{n1} is the first order perturbation in the energy for state n. The coefficients α and β determine the linear combination of the unperturbed wave functions according to

$$|n0\rangle=\alpha|a0\rangle+\beta|b0\rangle \quad (4)$$

The generalization is more easily seen if we rewrite the first pair of equations in matrix form

$$\left[\begin{array}{cc} W_{aa} & W_{ab}\\ W_{ba} & W_{bb} \end{array}\right]\left[\begin{array}{c} \alpha\\ \beta \end{array}\right]=E_{n1}\left[\begin{array}{c} \alpha\\ \beta \end{array}\right] \quad (5)$$

That is, the energy perturbations are the eigenvalues of the matrix W and the vectors $\left[\begin{array}{c} \alpha\\ \beta \end{array}\right]$ are the eigenvectors.

In the general case where we have a d-fold degeneracy with unperturbed energy E_{n0}, the most general unperturbed wave function is

$$|n0\rangle=\sum_{j=1}^{d}\alpha_{j}|j0\rangle \quad (6)$$

The derivation in the d-fold case is exactly the same as in the 2-fold case up until we get the equation

$$\langle a0|V|n0\rangle = E_{n1}\langle a0|n0\rangle \tag{7}$$

where $|a0\rangle$ is one of the degenerate unperturbed states. We now substitute for $|n0\rangle$ and using the orthogonality of the unperturbed states we get

$$\sum_{j=1}^{d}\alpha_j\langle a0|V|j0\rangle = \alpha_a E_{n1} \tag{8}$$

$$\sum_{j=1}^{d}\alpha_j W_{aj} = \alpha_a E_{n1} \tag{9}$$

This is just the ath row of the eigenvalue equation for the $d\times d$ matrix W so the eigenvalue equation applies to any level of degeneracy.

In systems that have some states with one level of degeneracy and other states with different levels of degeneracy, we would apply the appropriate eigenvalue equation to each set of degenerate states separately. For example, in the particle on a circular wire problem, all states except $n=0$ are doubly degenerate so we use the 2×2 matrix for each value of $n>0$ and the nondegenerate perturbation theory for $n=0$.

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