

ZEEMAN EFFECT: THE N=2 LINE IN HYDROGEN

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 6.23.

We'll show how the strong field Zeeman effect alters the spectrum of hydrogen for the case $n = 2$. The full formula for the energy levels is

$$(1) \quad E_n = -\frac{13.6 \text{ eV}}{n^2} \left[1 - \frac{\alpha^2}{n} \left(\frac{3}{4n} - \frac{\ell(\ell+1) - \ell_z s_z}{\ell(\ell+\frac{1}{2})(\ell+1)} \right) \right] + \mu_B (\ell_z + 2s_z) B_{ext}$$
$$(2) \quad = E_{2,0} + E_{fs1} + E_{Z1}$$

which is the sum of the Bohr energy, and the fine structure and the Zeeman corrections.

For $n = 2$, the Bohr energy is

$$(3) \quad E_{2,0} = -\frac{13.6 \text{ eV}}{2^2} = -3.4 \text{ eV}$$

and this is the same for all substates.

The fine structure term is

$$(4) \quad E_{fs1} = (3.4 \text{ eV}) \frac{\alpha^2}{2} \left(\frac{3}{8} - \frac{\ell(\ell+1) - \ell_z s_z}{\ell(\ell+\frac{1}{2})(\ell+1)} \right)$$

If $\ell = 0$, the second term in the parentheses is zero divided by zero (since the only possible value of ℓ_z is zero in this case), but as we'll see in the next post, we can take this quotient to be 1. For $\ell = 1$ we can just plug in the numbers and get

$$(5) \quad E_{fs1}(\ell = 1) = (3.4 \text{ eV}) \frac{\alpha^2}{2} \left(\frac{3}{8} - \frac{2 - \ell_z s_z}{3} \right)$$

. The Zeeman term depends only on the z components. The results are

ℓ	ℓ_z	s_z	$E_{Z1} (\times \mu_B B_{ext})$	$E_{fs1} (\times 3.4\alpha^2, \text{eV})$	E_n
0	0	$\frac{1}{2}$	1	$-\frac{5}{16} = -0.3125$	$-3.4 \left(1 + \frac{5}{16}\alpha^2\right) + \mu_B B_{ext}$
0	0	$-\frac{1}{2}$	-1	$-\frac{5}{16} = -0.3125$	$-3.4 \left(1 + \frac{5}{16}\alpha^2\right) - \mu_B B_{ext}$
1	-1	$\frac{1}{2}$	0	$-\frac{11}{48} = -0.2292$	$-3.4 \left(1 + \frac{11}{48}\alpha^2\right)$
1	-1	$-\frac{1}{2}$	-2	$-\frac{1}{16} = -0.0625$	$-3.4 \left(1 + \frac{1}{16}\alpha^2\right) - 2\mu_B B_{ext}$
1	0	$\frac{1}{2}$	1	$-\frac{7}{48} = -0.1458$	$-3.4 \left(1 + \frac{7}{48}\alpha^2\right) + \mu_B B_{ext}$
1	0	$-\frac{1}{2}$	-1	$-\frac{7}{48} = -0.1458$	$-3.4 \left(1 + \frac{7}{48}\alpha^2\right) - \mu_B B_{ext}$
1	1	$\frac{1}{2}$	2	$-\frac{1}{16} = -0.0625$	$-3.4 \left(1 + \frac{1}{16}\alpha^2\right) + 2\mu_B B_{ext}$
1	1	$-\frac{1}{2}$	0	$-\frac{11}{48} = -0.2292$	$-3.4 \left(1 + \frac{11}{48}\alpha^2\right)$

Looking only at the Zeeman energies, there are 5 distinct energies, 3 of which have degeneracy 2 and the other 2 of which have degeneracy 1. Two of the states are unaffected by the external magnetic field.

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