ZEEMAN EFFECT: THE N=2 LINE IN HYDROGEN

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 6.23.

We'll show how the strong field Zeeman effect alters the spectrum of hydrogen for the case $n=2$. The full formula for the energy levels is

(0.1)
$$E_n = -\frac{13.6\text{ eV}}{n^2}\left[1-\frac{\alpha^2}{n}\left(\frac{3}{4n}-\frac{\ell(\ell+1)-\ell_z s_z}{\ell\left(\ell+\frac{1}{2}\right)(\ell+1)}\right)\right]+\mu_B(\ell_z+2s_z)B_{ext}$$

(0.2)
$$= E_{2,0}+E_{fs1}+E_{Z1}$$

which is the sum of the Bohr energy, and the fine structure and the Zeeman corrections.

For $n=2$, the Bohr energy is

$$E_{2,0} = -\frac{13.6\text{ eV}}{2^2} = -3.4\text{ eV} \tag{0.3}$$

and this is the same for all substates.

The fine structure term is

$$E_{fs1} = (3.4\text{ eV})\frac{\alpha^2}{2}\left(\frac{3}{8}-\frac{\ell(\ell+1)-\ell_z s_z}{\ell\left(\ell+\frac{1}{2}\right)(\ell+1)}\right) \tag{0.4}$$

If $\ell=0$, the second term in the parentheses is zero divided by zero (since the only possible value of ℓ_z is zero in this case), but as we'll see in the next post, we can take this quotient to be 1. For $\ell=1$ we can just plug in the numbers and get

$$E_{fs1}(\ell=1) = (3.4\text{ eV})\frac{\alpha^2}{2}\left(\frac{3}{8}-\frac{2-\ell_z s_z}{3}\right) \tag{0.5}$$

. The Zeeman term depends only on the z components. The results are

ℓ	ℓ_z	s_z	$E_{Z1}\left(\times\mu_B B_{ext}\right)$	$E_{fs1}\left(\times 3.4\alpha^2,\text{ eV}\right)$	E_n
0	0	$\frac{1}{2}$	1	$-\frac{5}{16}=-0.3125$	$-3.4\left(1+\frac{5}{16}\alpha^2\right)+\mu_B B_{ext}$
0	0	$-\frac{1}{2}$	-1	$-\frac{5}{16}=-0.3125$	$-3.4\left(1+\frac{5}{16}\alpha^2\right)-\mu_B B_{ext}$
1	-1	$\frac{1}{2}$	0	$-\frac{11}{48}=-0.2292$	$-3.4\left(1+\frac{11}{48}\alpha^2\right)$
1	-1	$-\frac{1}{2}$	-2	$-\frac{1}{16}=-0.0625$	$-3.4\left(1+\frac{1}{16}\alpha^2\right)-2\mu_B B_{ext}$
1	0	$\frac{1}{2}$	1	$-\frac{7}{48}=-0.1458$	$-3.4\left(1+\frac{7}{48}\alpha^2\right)+\mu_B B_{ext}$
1	0	$-\frac{1}{2}$	-1	$-\frac{7}{48}=-0.1458$	$-3.4\left(1+\frac{7}{48}\alpha^2\right)-\mu_B B_{ext}$
1	1	$\frac{1}{2}$	2	$-\frac{1}{16}=-0.0625$	$-3.4\left(1+\frac{1}{16}\alpha^2\right)+2\mu_B B_{ext}$
1	1	$-\frac{1}{2}$	0	$-\frac{11}{48}=-0.2292$	$-3.4\left(1+\frac{11}{48}\alpha^2\right)$

Looking only at the Zeeman energies, there are 5 distinct energies, 3 of which have degeneracy 2 and the other 2 of which have degeneracy 1. Two of the states are unaffected by the external magnetic field.

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