In the strong field Zeeman effect, we used eigenstates of the Zeeman Hamiltonian to calculate the perturbations. This gave rise to the fine-structure correction
\[ E_{fs1,s} = \frac{13.6 \text{ eV}}{n^3} \alpha^2 \left( \frac{3}{4n} - \frac{\ell (\ell + 1) - \ell_z s_z}{\ell (\ell + \frac{1}{2}) (\ell + 1)} \right) \] (1)

We noted that the second term in parentheses is indeterminate if \( \ell = 0 \), so here we’ll address this point.

If \( \ell = 0 \), then the total angular momentum quantum number \( j \) becomes equal to the spin \( s = \frac{1}{2} \) and \( j_z = s_z \). The energy correction for the strong field Zeeman effect is then
\[ E_{Z1,s} = \mu_B (\ell_z + 2s_z) B_{ext} = 2\mu_B s_z B_{ext} \] (2)

and for the weak field it is
\[ E_{Z1,w} = \mu_B B_{ext} j_z \left( 1 + \frac{j (j + 1) + \frac{3}{4} - \ell (\ell + 1)}{2j (j + 1)} \right) = 2\mu_B s_z B_{ext} \] (3)

Thus the Zeeman corrections are the same for both types of field strength when \( \ell = 0 \).

The fine structure correction we used in the weak field case is
\[ E_{fs1,w} = \frac{13.6 \text{ eV}}{n^3} \alpha^2 \left( \frac{3}{4n} - \frac{1}{j + \frac{1}{2}} \right) \] (4)

With \( j = \frac{1}{2} \), this becomes
\[ E_{fs1,w} = \frac{13.6 \text{ eV}}{n^3} \alpha^2 \left( \frac{3}{4n} - 1 \right) \] (5)

This agrees with \( E_{fs1,s} \) at \( \ell = 0 \) if we take
ZEEMAN EFFECT FOR $L=0$

\[ \frac{\ell (\ell + 1) - \ell_z s_z}{\ell (\ell + \frac{1}{2})(\ell + 1)} \bigg|_{\ell=0} = 1 \]  

PINGBACKS

Pingback: Zeeman effect for $n = 3$: strong field
Pingback: Second-order correction to zeeman effect in hydrogen