ZEEMAN EFFECT FOR L=0

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 6.24.

In the strong field Zeeman effect, we used eigenstates of the Zeeman hamiltonian to calculate the perturbations. This gave rise to the fine-structure correction

$$E_{fs1,s}=\frac{13.6\text{ eV}}{n^{3}}\alpha^{2}\left[\frac{3}{4n}-\frac{\ell(\ell+1)-\ell_{z}s_{z}}{\ell\left(\ell+\frac{1}{2}\right)(\ell+1)}\right] \tag{1}$$

We noted that the second term in parentheses is indeterminate if $\ell=0$, so here we'll address this point.

If $\ell=0$, then the total angular momentum quantum number j becomes equal to the spin $s=\frac{1}{2}$ and $j_{z}=s_{z}$. The energy correction for the strong field Zeeman effect is then

$$E_{Z1,s}=\mu_{B}(\ell_{z}+2s_{z})B_{ext}=2\mu_{B}s_{z}B_{ext} \tag{2}$$

and for the weak field it is

$$E_{Z1,w}=\mu_{B}B_{ext}j_{z}\left(1+\frac{j(j+1)+\frac{3}{4}-\ell(\ell+1)}{2j(j+1)}\right)=2\mu_{B}s_{z}B_{ext} \tag{3}$$

Thus the Zeeman corrections are the same for both types of field strength when $\ell=0$.

The fine structure correction we used in the weak field case is

$$E_{fs1,w}=\frac{13.6\text{ eV}}{n^{3}}\alpha^{2}\left(\frac{3}{4n}-\frac{1}{j+\frac{1}{2}}\right) \tag{4}$$

With $j=\frac{1}{2}$, this becomes

$$E_{fs1,w}=\frac{13.6\text{ eV}}{n^{3}}\alpha^{2}\left(\frac{3}{4n}-1\right) \tag{5}$$

This agrees with $E_{fs1,s}$ at $\ell=0$ if we take

$$\left.\frac{\ell(\ell+1)-\ell_z s_z}{\ell\left(\ell+\frac{1}{2}\right)(\ell+1)}\right|_{\ell=0}=1 \tag{6}$$

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