ZEEMAN EFFECT FOR N = 3; WEAK FIELD

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 6.26 (weak field).

As a large-scale example of the Zeeman effect, we'll analyze the $n=3$ line of hydrogen. This isn't particularly difficult, but it is a lot of work so we'll break the analysis into three posts. In this post, we'll examine the weak field limit. In this case, the energies are given by the formula

$$E_n = -\frac{13.6\text{ eV}}{n^2}\left[1-\frac{\alpha^2}{4n^2}\left(3-\frac{4n}{j+\frac{1}{2}}\right)\right]+\mu_B g_J B_{ext} j_z \tag{1}$$

$$= -\frac{13.6\text{ eV}}{n^2}+E_{fs1}+E_{Z1} \tag{2}$$

with

$$E_{fs1} = \frac{13.6\text{ eV}}{n^2}\frac{\alpha^2}{4n^2}\left(3-\frac{4n}{j+\frac{1}{2}}\right) \tag{3}$$

$$E_{Z1} = \mu_B B_{ext} g_J j_z \equiv \beta g_J j_z \tag{4}$$

$$g_J \equiv 1+\frac{j(j+1)+\frac{3}{4}-\ell(\ell+1)}{2j(j+1)} \tag{5}$$

For $n=3$, we get

$$E_{fs1} = \frac{13.6\text{ eV}}{3^4\times 4}\alpha^2\left(3-\frac{12}{j+\frac{1}{2}}\right) \tag{6}$$

$$= \frac{13.6\text{ eV}}{108}\alpha^2\left(1-\frac{4}{j+\frac{1}{2}}\right) \tag{7}$$

$$\equiv \gamma\left(1-\frac{4}{j+\frac{1}{2}}\right) \tag{8}$$

The energies can be found by calculating all possible combinations of ℓ, j and j_z and working out the terms. We get

ℓ	j	j_z	g_J	$E_{fs1}+E_{Z1}$
0	$\frac{1}{2}$	$\frac{1}{2}$	2	$-3\gamma+\beta$
0	$\frac{1}{2}$	$-\frac{1}{2}$	2	$-3\gamma-\beta$
1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{3}$	$-3\gamma+\frac{1}{3}\beta$
1	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{2}{3}$	$-3\gamma-\frac{1}{3}\beta$
1	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{4}{3}$	$-\gamma+2\beta$
1	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{4}{3}$	$-\gamma+\frac{2}{3}\beta$
1	$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{4}{3}$	$-\gamma-\frac{2}{3}\beta$
1	$\frac{3}{2}$	$-\frac{3}{2}$	$\frac{4}{3}$	$-\gamma-2\beta$
2	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{4}{5}$	$-\gamma+\frac{6}{5}\beta$
2	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{4}{5}$	$-\gamma+\frac{2}{5}\beta$
2	$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{4}{5}$	$-\gamma-\frac{2}{5}\beta$
2	$\frac{3}{2}$	$-\frac{3}{2}$	$\frac{4}{5}$	$-\gamma-\frac{6}{5}\beta$
2	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{6}{5}$	$-\frac{1}{3}\gamma+3\beta$
2	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{6}{5}$	$-\frac{1}{3}\gamma+\frac{9}{5}\beta$
2	$\frac{5}{2}$	$\frac{1}{2}$	$\frac{6}{5}$	$-\frac{1}{3}\gamma+\frac{3}{5}\beta$
2	$\frac{5}{2}$	$-\frac{1}{2}$	$\frac{6}{5}$	$-\frac{1}{3}\gamma-\frac{3}{5}\beta$
2	$\frac{5}{2}$	$-\frac{3}{2}$	$\frac{6}{5}$	$-\frac{1}{3}\gamma-\frac{9}{5}\beta$
2	$\frac{5}{2}$	$-\frac{5}{2}$	$\frac{6}{5}$	$-\frac{1}{3}\gamma-3\beta$

The total energy for each level is

$$E_{n1} = -\frac{13.6\text{ eV}}{9} + E_{fs1} + E_{Z1} \tag{9}$$

$$= -1.51\text{ eV} + E_{fs1} + E_{Z1} \tag{10}$$

There are 18 distinct energy levels.

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