

## ZEEMAN EFFECT FOR N = 3; WEAK FIELD

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 6.26 (weak field).

As a large-scale example of the Zeeman effect, we'll analyze the  $n = 3$  line of hydrogen. This isn't particularly difficult, but it is a lot of work so we'll break the analysis into three posts. In this post, we'll examine the weak field limit. In this case, the energies are given by the formula

$$E_n = -\frac{13.6 \text{ eV}}{n^2} \left[ 1 - \frac{\alpha^2}{4n^2} \left( 3 - \frac{4n}{j + \frac{1}{2}} \right) \right] + \mu_B g_J B_{ext} j_z \quad (1)$$

$$= -\frac{13.6 \text{ eV}}{n^2} + E_{fs1} + E_{Z1} \quad (2)$$

with

$$E_{fs1} = \frac{13.6 \text{ eV}}{n^2} \frac{\alpha^2}{4n^2} \left( 3 - \frac{4n}{j + \frac{1}{2}} \right) \quad (3)$$

$$E_{Z1} = \mu_B B_{ext} g_J j_z \equiv \beta g_J j_z \quad (4)$$

$$g_J \equiv 1 + \frac{j(j+1) + \frac{3}{4} - \ell(\ell+1)}{2j(j+1)} \quad (5)$$

For  $n = 3$ , we get

$$E_{fs1} = \frac{13.6 \text{ eV}}{3^4 \times 4} \alpha^2 \left( 3 - \frac{12}{j + \frac{1}{2}} \right) \quad (6)$$

$$= \frac{13.6 \text{ eV}}{108} \alpha^2 \left( 1 - \frac{4}{j + \frac{1}{2}} \right) \quad (7)$$

$$\equiv \gamma \left( 1 - \frac{4}{j + \frac{1}{2}} \right) \quad (8)$$

The energies can be found by calculating all possible combinations of  $\ell$ ,  $j$  and  $j_z$  and working out the terms. We get

$\ell$	$j$	$j_z$	$gJ$	$E_{fs1} + E_{Z1}$
0	$\frac{1}{2}$	$\frac{1}{2}$	2	$-3\gamma + \beta$
0	$\frac{1}{2}$	$-\frac{1}{2}$	2	$-3\gamma - \beta$
1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{3}$	$-3\gamma + \frac{1}{3}\beta$
1	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{2}{3}$	$-3\gamma - \frac{1}{3}\beta$
1	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{4}{3}$	$-\gamma + 2\beta$
1	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{4}{3}$	$-\gamma + \frac{2}{3}\beta$
1	$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{4}{3}$	$-\gamma - \frac{2}{3}\beta$
1	$\frac{3}{2}$	$-\frac{3}{2}$	$\frac{4}{3}$	$-\gamma - 2\beta$
2	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{4}{5}$	$-\gamma + \frac{6}{5}\beta$
2	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{4}{5}$	$-\gamma + \frac{2}{5}\beta$
2	$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{4}{5}$	$-\gamma - \frac{2}{5}\beta$
2	$\frac{3}{2}$	$-\frac{3}{2}$	$\frac{4}{5}$	$-\gamma - \frac{6}{5}\beta$
2	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{6}{5}$	$-\frac{1}{3}\gamma + 3\beta$
2	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{6}{5}$	$-\frac{1}{3}\gamma + \frac{9}{5}\beta$
2	$\frac{5}{2}$	$\frac{1}{2}$	$\frac{6}{5}$	$-\frac{1}{3}\gamma + \frac{3}{5}\beta$
2	$\frac{5}{2}$	$-\frac{1}{2}$	$\frac{6}{5}$	$-\frac{1}{3}\gamma - \frac{3}{5}\beta$
2	$\frac{5}{2}$	$-\frac{3}{2}$	$\frac{6}{5}$	$-\frac{1}{3}\gamma - \frac{9}{5}\beta$
2	$\frac{5}{2}$	$-\frac{5}{2}$	$\frac{6}{5}$	$-\frac{1}{3}\gamma - 3\beta$

The total energy for each level is

$$E_{n1} = -\frac{13.6 \text{ eV}}{9} + E_{fs1} + E_{Z1} \quad (9)$$

$$= -1.51 \text{ eV} + E_{fs1} + E_{Z1} \quad (10)$$

There are 18 distinct energy levels.

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