HYPERFINE SPLITTING IN DEUTERIUM

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 6.38.

We can repeat the calculation done earlier to determine the hyperfine splitting in the hydrogen atom, only this time we'll consider deuterium, where the nucleus consists of 1 proton and 1 neutron, with total spin 1. The calculation is the same as before, except references to the proton are replaced with those to the deuteron (the proton-neutron pair). Thus we get

$$E_{hs1} = \frac{\mu_0 e^2 g_d}{3m_d m_e}\left\langle \delta^3(\mathbf{r})\right\rangle \left\langle \mathbf{S}_d \cdot \mathbf{S}_e \right\rangle \tag{1}$$

The deuteron's g-factor is $g_d = 1.71$ and the mass of the deuteron is roughly $m_d = 2m_p$. The ground state wave function is the same so $\left\langle \delta^3(\mathbf{r})\right\rangle = |\psi_{100}(0)|^2 = 1/\pi a^3$ as before. The only thing left to find is $\left\langle \mathbf{S}_d \cdot \mathbf{S}_e \right\rangle$. As before, we can write this as

$$\mathbf{S}_d \cdot \mathbf{S}_e = \frac{1}{2}\left(S^2 - S_e^2 - S_d^2\right) \tag{2}$$

$$= \frac{1}{2}\left(S^2 - \frac{3}{4}\hbar^2 - 2\hbar^2\right) \tag{3}$$

The two possible values of s are $\frac{1}{2}$ and $\frac{3}{2}$, so $S^2 = \frac{3}{4}\hbar^2$ or $\frac{15}{4}\hbar^2$ and $\left\langle \mathbf{S}_d \cdot \mathbf{S}_e \right\rangle = \frac{1}{2}\hbar^2$ or $-\hbar^2$. The two energies are therefore

$$E_{hs1} = \begin{cases} \frac{1}{2}\frac{\mu_0 e^2 g_d \hbar^2}{3\pi a^3 m_d m_e} & \left(s = \frac{3}{2}\right) \\ -\frac{\mu_0 e^2 g_d \hbar^2}{3\pi a^3 m_d m_e} & \left(s = \frac{1}{2}\right) \end{cases} \tag{4}$$

After multiplying top and bottom by the Bohr radius $a = 4\pi\epsilon_0\hbar^2/m_e e^2$ and using the relation $\epsilon_0\mu_0 = 1/c^2$ we get the difference between the two energies as

$$\Delta E = \frac{3}{2}\frac{4g_d\hbar^4}{3m_d m_e^2 c^2 a^4} = 1.349\times 10^{-6}\text{ eV} \tag{5}$$

This corresponds to a frequency of

$$\nu = \frac{\Delta E}{h} = 325.9 \text{ MHz} \tag{6}$$

with a wavelength of

$$\lambda = \frac{c}{\nu} = 92 \text{ cm} \tag{7}$$