

## VARIATIONAL PRINCIPLE AND THE ELECTRON IN A MAGNETIC FIELD

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 7.16.

As an example of the two-state system we looked at earlier, we can consider an electron in a magnetic field. The hamiltonian for a spin-1/2 particle such as the electron in a magnetic field is

$$(1) \quad H = -\gamma \mathbf{B} \cdot \mathbf{S}$$

where  $\mathbf{S}$  is the spin matrix and  $\gamma = -e/m$  is the gyromagnetic ratio of the electron.

We start with a uniform magnetic field in the  $z$  direction  $\mathbf{B} = B_z \hat{\mathbf{z}}$ , for which the hamiltonian is

$$(2) \quad H = \frac{e}{m} B_z S_z = \frac{e B_z \hbar}{2m} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

The energies are

$$(3) \quad E_{b,a} = \pm \frac{e B_z \hbar}{2m}$$

(taking  $E_b > E_a$  as before).

We now introduce a perturbation by turning on a weak field in the  $x$  direction  $\mathbf{B}' = B_x \hat{\mathbf{x}}$ . The perturbation in the hamiltonian is

$$(4) \quad H' = \frac{e B_x \hbar}{2m} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

This has the same form as the perturbation in the previous problem:

$$(5) \quad H' = \begin{bmatrix} 0 & h \\ h & 0 \end{bmatrix}$$

with

$$(6) \quad h = \frac{eB_x \hbar}{2m}$$

Using second order perturbation theory, the the perturbations on the two energies are

$$(7) \quad E_{a2} = \frac{h^2}{E_a - E_b} = -\frac{\hbar e B_x^2}{4m B_z}$$

$$(8) \quad E_{b2} = \frac{h^2}{E_b - E_a} = \frac{\hbar e B_x^2}{4m B_z}$$

The variational calculation gave the exact answer, which is

$$(9) \quad \langle H \rangle_{min} = \frac{1}{2} \left( E_a + E_b - \sqrt{(E_a - E_b)^2 + 4h^2} \right)$$

$$(10) \quad = \frac{\hbar e}{2m} \sqrt{B_x^2 + B_z^2}$$

That is, it is the same as 3 except we now have the magnitude of the full magnetic field.