SPONTANEOUS EMISSION FROM THE ZERO POINT FIELD

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 9.9.

Viewed in quantum electrodynamics, spontaneous emission of radiation is actually emission stimulated by the zero point electromagnetic field. The transition rate for stimulated emission is given by Griffiths (his equation 9.47) and is

$$R_{a\to b} = \frac{\pi\left|\mathfrak{p}\right|^2}{3\epsilon_0\hbar^2}\rho\left(\omega_0\right) \tag{1}$$

where

$$\mathfrak{p} \equiv q\left\langle a\left|\mathbf{r}\right|b\right\rangle \tag{2}$$

is the dipole moment averaged between the two states a and b. If the stimulating radiation is due to thermal processes, we can use Planck's formula for $\rho\left(\omega_0\right)$:

$$\rho\left(\omega_0\right) = \frac{1}{\left(e^{\hbar\omega_0/k_BT}-1\right)}\frac{\hbar\omega_0^3}{\pi^2c^3} \tag{3}$$

However, for the zero point field, Planck's formula does not apply. To get the formula properly, we'd need to use quantum electrodynamics, but following Griffiths, we can assume that the number of photons per available state is 1. That is, given that the number of states in the shell from wave number k to $k+dk$ is (using $k=\omega/c$):

$$d_k = \frac{k^2V}{\pi^2}dk \tag{4}$$

$$= \frac{\omega^2V}{\pi^2c^2}d\omega \tag{5}$$

where V is the volume of the box in which we put the photons, then the number of photons in that range of wave numbers is just

$$N_\omega = d_k \tag{6}$$

rather than the formula we had used earlier: $N_\omega = \frac{d_k}{e^{\hbar\omega/k_B T}-1}$.

The energy density is found by using the fact that the energy of a photon is $h\nu = \hbar\omega$, so $\rho(\omega)\, d\omega = N_\omega \hbar\omega / V$, so

$$\rho(\omega)\, d\omega = \frac{d_k \hbar\omega}{V} \tag{7}$$

$$= \frac{\hbar\omega^3}{\pi^2 c^2} d\omega \tag{8}$$

For the resonant frequency $\omega_0 = (E_b - E_a)/\hbar$ we thus have

$$\rho(\omega_0) = \frac{\hbar\omega_0^3}{\pi^2 c^2} \tag{9}$$

so from 1 we have

$$R_{a\to b} = \frac{\omega_0^3 |\mathfrak{p}|^2}{3\pi\epsilon_0 \hbar c^3} \tag{10}$$

which is the same result as we got using Einstein's method.