

**SELECTION RULES IN SPONTANEOUS EMISSION:
TRANSITION BETWEEN SPHERICALLY SYMMETRIC STATES
NOT ALLOWED**

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 9.13.

When deriving the selection rules for states connected by spontaneous emission of a photon in systems with a spherically symmetric potential we found that the l quantum numbers of the starting and ending states had to be related by

$$(1) \quad \left[(l' + l + 1)^2 - 1 \right] \left[(l' - l)^2 - 1 \right] = 0$$

One solution is $l' = l \pm 1$ (which makes the second factor 0), but the solution $l' = l = 0$ also seems acceptable, since it makes the first factor 0. However, in the latter case we must have $m' = m = 0$ (since the quantum number m must satisfy $-l \leq m \leq +l$) so all matrix elements in the transition rate formula must be of the form

$$(2) \quad \langle \psi_b | \mathbf{r} | \psi_a \rangle = \langle n'00 | \mathbf{r} | n00 \rangle$$

When $l = m = 0$, the angular part of the wave function is given by the spherical harmonic

$$(3) \quad Y_0^0 = \frac{1}{\sqrt{4\pi}}$$

so the angular integrals in 2 are integrals over x , y and z expressed in spherical coordinates (the wave functions are both independent of the angular coordinates θ and ϕ). We therefore have for the angular integrals of the $x = r \sin \theta \cos \phi$ component

$$(4) \quad \int_0^\pi \int_0^{2\pi} \sin^2 \theta \cos \phi d\phi d\theta = 0$$

For $y = r \sin \theta \sin \phi$ we get

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$$(5) \quad \int_0^\pi \int_0^{2\pi} \sin^2 \theta \sin \phi d\phi d\theta = 0$$

and for $z = r \cos \theta$ we get

$$(6) \quad \int_0^\pi \int_0^{2\pi} \sin \theta \cos \theta d\phi d\theta = 0$$

Therefore the dipole moment matrix element is identically zero if $l' = l = 0$:

$$(7) \quad \mathbf{p} = q \langle \psi_b | \mathbf{r} | \psi_a \rangle = 0$$

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