SELECTION RULES IN SPONTANEOUS EMISSION:
TRANSITION BETWEEN SPHERICALLY SYMMETRIC STATES
NOT ALLOWED

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When deriving the selection rules for states connected by spontaneous emission of a photon in systems with a spherically symmetric potential we found that the \( l \) quantum numbers of the starting and ending states had to be related by

\[
\left( (l' + l + 1)^2 - 1 \right) \left( (l' - l)^2 - 1 \right) = 0 \tag{1}
\]

One solution is \( l' = l \pm 1 \) (which makes the second factor 0), but the solution \( l' = l = 0 \) also seems acceptable, since it makes the first factor 0. However, in the latter case we must have \( m' = m = 0 \) (since the quantum number \( m \) must satisfy \(-l \leq m \leq +l\)) so all matrix elements in the transition rate formula must be of the form

\[
\langle \psi_b | r | \psi_a \rangle = \langle n'00 | r | n00 \rangle \tag{2}
\]

When \( l = m = 0 \), the angular part of the wave function is given by the spherical harmonic

\[
Y_0^0 = \frac{1}{\sqrt{4\pi}} \tag{3}
\]

so the angular integrals in \( \psi_b \) are integrals over \( x, y \) and \( z \) expressed in spherical coordinates (the wave functions are both independent of the angular coordinates \( \theta \) and \( \phi \)). We therefore have for the angular integrals of the \( x = r \sin \theta \cos \phi \) component

\[
\int_0^\pi \int_0^{2\pi} \sin^2 \theta \cos \phi d\phi d\theta = 0 \tag{4}
\]

For \( y = r \sin \theta \sin \phi \) we get

\[
\int_0^\pi \int_0^{2\pi} \sin^2 \theta \sin \phi d\phi d\theta = 0 \tag{5}
\]
and for $z = r \cos \theta$ we get

$$\int_0^\pi \int_0^{2\pi} \sin \theta \cos \theta d\phi d\theta = 0$$

Therefore the dipole moment matrix element is identically zero if $l' = l = 0$:

$$p = q \langle \psi_b | r | \psi_a \rangle = 0$$

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