SELECTION RULES IN SPONTANEOUS EMISSION: TRANSITION BETWEEN SPHERICALLY SYMMETRIC STATES NOT ALLOWED

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 9.13.

When deriving the selection rules for states connected by spontaneous emission of a photon in systems with a spherically symmetric potential we found that the l quantum numbers of the starting and ending states had to be related by

$$\left[\left(l'+l+1\right)^2-1\right]\left[\left(l'-l\right)^2-1\right]=0 \tag{0.1}$$

One solution is $l'=l\pm1$ (which makes the second factor 0), but the solution $l'=l=0$ also seems acceptable, since it makes the first factor 0. However, in the latter case we must have $m'=m=0$ (since the quantum number m must satisfy $-l\le m\le+l$) so all matrix elements in the transition rate formula must be of the form

$$\left\langle \psi_b\left|\mathbf{r}\right|\psi_a\right\rangle=\left\langle n'00\left|\mathbf{r}\right|n00\right\rangle \tag{0.2}$$

When $l=m=0$, the angular part of the wave function is given by the spherical harmonic

$$Y_0^0=\frac{1}{\sqrt{4\pi}} \tag{0.3}$$

so the angular integrals in 0.2 are integrals over x, y and z expressed in spherical coordinates (the wave functions are both independent of the angular coordinates θ and ϕ). We therefore have for the angular integrals of the $x=r\sin\theta\cos\phi$ component

$$\int_0^{\pi}\int_0^{2\pi}\sin^2\theta\cos\phi\, d\phi\, d\theta=0 \tag{0.4}$$

For $y=r\sin\theta\sin\phi$ we get

$$\int_0^{\pi}\int_0^{2\pi}\sin^2\theta\sin\phi d\phi d\theta=0 \tag{0.5}$$

and for $z=r\cos\theta$ we get

$$\int_0^{\pi}\int_0^{2\pi}\sin\theta\cos\theta d\phi d\theta=0 \tag{0.6}$$

Therefore the dipole moment matrix element is identically zero if $l'=l=0$:

$$\mathfrak{p}=q\left\langle\psi_b\left|\mathbf{r}\right|\psi_a\right\rangle=0 \tag{0.7}$$

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