zeitabhängige Störungstheorie: Übergangswahrscheinlichkeiten in einem multi-staatem System

Link zu: physicspages home page.

Zum hinterlassen eines Kommentars oder der Meldestelle eines Fehlers, bitte den auxiliary blog verwenden.


Postdate: 5 Nov 2014.

In time-dependent perturbation theory for a multi-state system we got a general expression for the coefficient $c_m(t)$ to first order. If the system starts in state $N$ then for $m \neq N$:

$$c_m(t) = -\frac{i}{\hbar} \int_0^t H'_{mN} e^{i(E_m - E_N)t'}/\hbar dt'$$ (1)

and

$$c_N^{(1)} = 1 - \frac{i}{\hbar} \int_0^t H'_{NN}(t') dt'$$ (2)

We looked at two examples of a perturbation: when the perturbation is constant between the times $t = 0$ and $t$, and where the perturbation is sinusoidal in the same interval. For the constant perturbation, we get

$$c_m^{(1)}(t) = -\frac{2iH'_{mN}}{(E_m - E_N)} e^{i(E_m - E_N)t/2\hbar} \sin \frac{(E_m - E_N)t}{2\hbar}$$ (3)

$$c_N^{(1)}(t) = 1 - \frac{i}{\hbar} H'_{NN} t$$ (4)

Since $H'$ is hermitian, the matrix elements $H'_{ij}$ are real, so the sum of all the transition probabilities is (we’ll drop the superscript (1) from now on, as we’re always dealing with the first order coefficients):

$$\sum_m |c_m(t)|^2 = 1 + \frac{|H'_{NN}|^2 t^2}{\hbar^2} + \sum_{m \neq N} |c_m(t)|^2$$

$$= 1 + \frac{|H'_{NN}|^2 t^2}{\hbar^2} + 4 \sum_{m \neq N} \frac{|H'_{mN}|^2}{(E_m - E_N)^2} \sin^2 \frac{(E_m - E_N)t}{2\hbar}$$ (6)
This is clearly greater than 1, but all the extra terms are second order in the matrix elements of the perturbation $H'_{ij}$, so to first order, we’re safe.

For the sinusoidal perturbation $H'(t) = V \cos (\omega t)$, we got

$$c_{m \neq N}(t) = -V_{mN} t e^{i(E_m - E_N \pm \hbar \omega)t/2\hbar} \frac{E_N - E_m \pm \hbar \omega}{-2\hbar} \sin \frac{(E_N - E_m \pm \hbar \omega)t}{2\hbar}$$

$$c_N(t) = 1 - \frac{i}{\hbar} V_{NN} \int_0^t \cos (\omega t') dt'$$

$$= 1 - \frac{i}{\hbar} V_{NN} \sin \omega t$$

The sum of all the transition probabilities is

$$\sum_m |c_m(t)|^2 = 1 + \frac{|V_{NN}|^2}{\hbar^2 \omega^2} \sin^2 \omega t + \sum_{m \neq N} |c_m(t)|^2$$

$$= 1 + \frac{|V_{NN}|^2}{\hbar^2 \omega^2} \sin^2 \omega t + \sum_{m \neq N} \frac{|V_{mN}|^2}{(E_N - E_m \pm \hbar \omega)^2} \sin^2 \frac{(E_N - E_m \pm \hbar \omega)t}{2\hbar}$$

Again, the sum is greater than 1 but all the offending terms are of second order.

To calculate the probability of the system remaining in state $N$ it would seem better to use $1 - \sum_{m \neq N} |c_m(t)|^2$ rather than $|c_N(t)|^2$ since the latter is greater than 1.