TIME-DEPENDENT PERTURBATION UNIFORM IN SPACE

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 9.17.

In a multi-state time dependent system (such as the infinite square well) suppose we introduce a perturbation $H' = V_0(t)$ which is a function of time only. That is, V_0 has the same value for all spatial locations at any given time t. We'll impose the boundary conditions $V_0(0) = V_0(T) = 0$ and start the system off in state ψ_N. The exact ODEs for this system are

$$\dot{c}_m = -\frac{i}{\hbar}\sum_n c_n(t) H'_{mn} e^{i(E_m - E_n)t/\hbar} \tag{1}$$

Since $H' = V_0(t)$ is independent of x and the basis states of the system are orthonormal, the matrix elements are

$$H'_{mn} = V_0(t)\,\delta_{mn} \tag{2}$$

For state N in particular, we get

$$\begin{aligned} \dot{c}_N &= -\frac{i}{\hbar} c_N V_0 && (3) \\ c_N(t) &= K e^{-i\int_0^t V_0(t')dt'/\hbar} && (4) \end{aligned}$$

where K is determined by the initial condition that $c_N(0) = 1$ so $K = 1$. For $c_{m\neq N}$ we get the same differential equation except that the initial condition is now $c_m(0) = 0$ so $K = 0$ and all c_ms remain zero, so no transitions occur. The complete wave function is then

$$\begin{aligned} \Psi(t) &= \sum_n c_n(t)\,\psi_n e^{-iE_n t/\hbar} && (5) \\ &= c_N(t)\,\psi_N e^{-iE_n t/\hbar} && (6) \\ &= \psi_N e^{-iE_n t/\hbar} e^{-i\int_0^t V_0(t')dt'/\hbar} && (7) \end{aligned}$$

The wave function is shifted by a phase

$$\phi(t) = -\frac{1}{\hbar}\int_0^t V_0(t')\,dt' \tag{8}$$

In first order perturbation theory, we get

$$c_N^{(1)} = 1-\frac{i}{\hbar}\int_0^t H'_{NN}\left(t'\right)dt' \tag{9}$$

$$= 1-\frac{i}{\hbar}\int_0^t V_0\left(t'\right)dt' \tag{10}$$

This is just the first-order Taylor expansion of the exponential in 4. The first order values of the other coefficients is

$$c_{m\neq N}\left(t\right) = -\frac{i}{\hbar}\int_0^t H'_{mN}e^{i(E_m-E_N)t'/\hbar}dt' = 0 \tag{11}$$

since $H'_{mN}=0$ if $m\neq N$. The answers from the two methods agree to first order.