

TIME-DEPENDENT PERTURBATION UNIFORM IN SPACE

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 9.17.

In a multi-state time dependent system (such as the infinite square well) suppose we introduce a perturbation $H' = V_0(t)$ which is a function of time only. That is, V_0 has the same value for all spatial locations at any given time t . We'll impose the boundary conditions $V_0(0) = V_0(T) = 0$ and start the system off in state ψ_N . The exact ODEs for this system are

$$(0.1) \quad \dot{c}_m = -\frac{i}{\hbar} \sum_n c_n(t) H'_{mn} e^{i(E_m - E_n)t/\hbar}$$

Since $H' = V_0(t)$ is independent of x and the basis states of the system are orthonormal, the matrix elements are

$$(0.2) \quad H'_{mn} = V_0(t) \delta_{mn}$$

For state N in particular, we get

$$(0.3) \quad \dot{c}_N = -\frac{i}{\hbar} c_N V_0$$

$$(0.4) \quad c_N(t) = K e^{-i \int_0^t V_0(t') dt' / \hbar}$$

where K is determined by the initial condition that $c_N(0) = 1$ so $K = 1$. For $c_{m \neq N}$ we get the same differential equation except that the initial condition is now $c_m(0) = 0$ so $K = 0$ and all c_m s remain zero, so no transitions occur. The complete wave function is then

$$(0.5) \quad \Psi(t) = \sum_n c_n(t) \psi_n e^{-iE_n t/\hbar}$$

$$(0.6) \quad = c_N(t) \psi_N e^{-iE_N t/\hbar}$$

$$(0.7) \quad = \psi_N e^{-iE_N t/\hbar} e^{-i \int_0^t V_0(t') dt' / \hbar}$$

The wave function is shifted by a phase

$$(0.8) \quad \phi(t) = -\frac{1}{\hbar} \int_0^t V_0(t') dt'$$

In first order perturbation theory, we get

$$(0.9) \quad c_N^{(1)} = 1 - \frac{i}{\hbar} \int_0^t H'_{NN}(t') dt'$$

$$(0.10) \quad = 1 - \frac{i}{\hbar} \int_0^t V_0(t') dt'$$

This is just the first-order Taylor expansion of the exponential in 0.4. The first order values of the other coefficients is

$$(0.11) \quad c_{m \neq N}(t) = -\frac{i}{\hbar} \int_0^t H'_{mN} e^{i(E_m - E_N)t'/\hbar} dt' = 0$$

since $H'_{mN} = 0$ if $m \neq N$. The answers from the two methods agree to first order.