TIME-DEPENDENT PERTURBATION UNIFORM IN SPACE

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 9.17.

In a multi-state time dependent system (such as the infinite square well) suppose we introduce a perturbation $H'=V_0(t)$ which is a function of time only. That is, V_0 has the same value for all spatial locations at any given time t. We'll impose the boundary conditions $V_0(0)=V_0(T)=0$ and start the system off in state ψ_N. The exact ODEs for this system are

$$\dot{c}_m=-\frac{i}{\hbar}\sum_n c_n(t)\,H'_{mn}e^{i(E_m-E_n)t/\hbar} \tag{1}$$

Since $H'=V_0(t)$ is independent of x and the basis states of the system are orthonormal, the matrix elements are

$$H'_{mn}=V_0(t)\,\delta_{mn} \tag{2}$$

For state N in particular, we get

$$\dot{c}_N = -\frac{i}{\hbar}c_N V_0 \tag{3}$$

$$c_N(t) = Ke^{-i\int_0^t V_0(t')dt'/\hbar} \tag{4}$$

where K is determined by the initial condition that $c_N(0)=1$ so $K=1$. For $c_{m\neq N}$ we get the same differential equation except that the initial condition is now $c_m(0)=0$ so $K=0$ and all c_ms remain zero, so no transitions occur. The complete wave function is then

$$\Psi(t) = \sum_n c_n(t)\,\psi_n e^{-iE_nt/\hbar} \tag{5}$$

$$= c_N(t)\,\psi_N e^{-iE_nt/\hbar} \tag{6}$$

$$= \psi_N e^{-iE_nt/\hbar}e^{-i\int_0^t V_0(t')dt'/\hbar} \tag{7}$$

The wave function is shifted by a phase

$$\phi(t) = -\frac{1}{\hbar}\int_0^t V_0(t')\,dt' \tag{8}$$

In first order perturbation theory, we get

$$c_N^{(1)} = 1 - \frac{i}{\hbar}\int_0^t H'_{NN}(t')\,dt' \tag{9}$$

$$= 1 - \frac{i}{\hbar}\int_0^t V_0(t')\,dt' \tag{10}$$

This is just the first-order Taylor expansion of the exponential in 4. The first order values of the other coefficients is

$$c_{m\neq N}(t) = -\frac{i}{\hbar}\int_0^t H'_{mN}e^{i(E_m-E_N)t'/\hbar}dt' = 0 \tag{11}$$

since $H'_{mN} = 0$ if $m \neq N$. The answers from the two methods agree to first order.