

TIME-DEPENDENT PERTURBATION OF THE INFINITE SQUARE WELL

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 9.18.

Here's another example of a perturbation in a multi-state time dependent system. We start with a particle of mass m in the ground state ψ_1 of the one dimensional infinite square well. At time $t = 0$ the potential is changed to

$$(1) \quad V(x) = \begin{cases} V_0 & 0 \leq x \leq \frac{a}{2} \\ 0 & \frac{a}{2} < x < a \\ \infty & \text{otherwise} \end{cases}$$

and at time $t = T$ the potential reverts to the unperturbed square well. We'd like to find the probability that the particle makes a transition to state ψ_2 .

To do this, we can use first-order perturbation theory to find the coefficient $c_2(T)$ in the wave function

$$(2) \quad \Psi(t) = \sum_n c_n(t) \psi_n e^{-iE_n t/\hbar}$$

The first order coefficient is given by

$$(3) \quad c_2(t) = -\frac{i}{\hbar} \int_0^T H'_{12} e^{i(E_2 - E_1)t/\hbar} dt$$

To get the matrix element H'_{12} we need the unperturbed wave functions:

$$(4) \quad \psi_n(x) = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}$$

We get

$$(5) \quad H'_{12} = \frac{2V_0}{a} \int_0^{a/2} \sin \frac{\pi x}{a} \sin \frac{2\pi x}{a} dx$$

$$(6) \quad = \frac{4V_0}{3\pi}$$

$$(7) \quad c_2 = \frac{4V_0}{3\pi} \frac{e^{i(E_2-E_1)T/\hbar} - 1}{E_2 - E_1}$$

$$(8) \quad = \frac{8iV_0}{3\pi(E_2 - E_1)} e^{i(E_2-E_1)T/2\hbar} \sin \frac{(E_2 - E_1)T}{2\hbar}$$

Using the energy levels of the infinite square well

$$(9) \quad E_n = \frac{(n\pi\hbar)^2}{2ma^2}$$

we get

$$(10) \quad E_2 - E_1 = \frac{3\pi^2\hbar^2}{2ma^2}$$

$$(11) \quad |c_2(T)|^2 = \left(\frac{16ma^2V_0}{9\pi^3\hbar^2} \right)^2 \sin^2 \left(\frac{3\pi^2\hbar T}{4ma^2} \right)$$