TIME-DEPENDENT PERTURBATION OF THE INFINITE SQUARE WELL

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 9.18.

Here's another example of a perturbation in a multi-state time dependent system. We start with a particle of mass m in the ground state ψ_1 of the one dimensional infinite square well. At time $t=0$ the potential is changed to

$$V(x)=\begin{cases}V_0 & 0\le x\le\frac{a}{2}\\ 0 & \frac{a}{2}<x<a\\ \infty & \text{otherwise}\end{cases} \tag{1}$$

and at time $t=T$ the potential reverts to the unperturbed square well. We'd like to find the probability that the particle makes a transition to state ψ_2.

To do this, we can use first-order perturbation theory to find the coefficient $c_2(T)$ in the wave function

$$\Psi(t)=\sum_n c_n(t)\,\psi_n e^{-iE_nt/\hbar} \tag{2}$$

The first order coefficient is given by

$$c_2(t)=-\frac{i}{\hbar}\int_0^T H'_{12}e^{i(E_2-E_1)t/\hbar}dt \tag{3}$$

To get the matrix element H'_{12} we need the unperturbed wave functions:

$$\psi_n(x)=\sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a} \tag{4}$$

We get

$$H'_{12} = \frac{2V_0}{a}\int_0^{a/2}\sin\frac{\pi x}{a}\sin\frac{2\pi x}{a}dx \tag{5}$$
$$= \frac{4V_0}{3\pi} \tag{6}$$
$$c_2 = \frac{4V_0}{3\pi}\frac{e^{i(E_2-E_1)T/\hbar}-1}{E_2-E_1} \tag{7}$$
$$= \frac{8iV_0}{3\pi\left(E_2-E_1\right)}e^{i(E_2-E_1)T/2\hbar}\sin\frac{\left(E_2-E_2\right)T}{2\hbar} \tag{8}$$

Using the energy levels of the infinite square well

$$E_n = \frac{\left(n\pi\hbar\right)^2}{2ma^2} \tag{9}$$

we get

$$E_2-E_1 = \frac{3\pi^2\hbar^2}{2ma^2} \tag{10}$$
$$\left|c_2\left(T\right)\right|^2 = \left(\frac{16ma^2V_0}{9\pi^3\hbar^2}\right)^2\sin^2\left(\frac{3\pi^2\hbar T}{4ma^2}\right) \tag{11}$$