

## PHASES IN THE ADIABATIC THEOREM: DELTA FUNCTION WELL

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 10.4.

Here's another example of calculating phases in the adiabatic theorem which says that if a system starts out in the  $n$ th state of a time-dependent hamiltonian, and the hamiltonian changes slowly compared to the internal period of the time-independent wave function (that is, the time scale over which the hamiltonian changes is much longer than  $\hbar/E_n$ ), then after a time  $t$  the system will end up in state

$$\Psi_n(t) = e^{i\theta_n(t)} e^{i\gamma_n(t)} \psi_n(t) \quad (1)$$

where

$$\theta_n(t) \equiv -\frac{1}{\hbar} \int_0^t E_n(t') dt' \quad (2)$$

$$\gamma_n(t) \equiv i \int_0^t \left\langle \psi_n(t') \left| \frac{\partial}{\partial t'} \psi_n(t') \right. \right\rangle dt' \quad (3)$$

$\theta$  is called the *dynamic phase* and  $\gamma$  is called the *geometric phase*.

The wave functions  $\psi_n(t)$  are the solutions of the eigenvalue equation at a particular time  $t$ :

$$H(t) \psi_n(t) = E_n(t) \psi_n(t) \quad (4)$$

That is, they aren't a full solution of the time dependent Schrödinger equation; rather they are the solutions of the time-independent Schrödinger equation with whatever parameters are now time-dependent in the hamiltonian replaced by their time-dependent forms.

With a delta function well the potential is

$$V(x) = -\alpha \delta(x) \quad (5)$$

and the time-independent wave function for the bound state is

$$\psi(x) = \frac{\sqrt{m\alpha}}{\hbar} e^{-m\alpha|x|/\hbar^2} \quad (6)$$

If the strength of the delta function  $\alpha$  varies with time, then

$$\gamma_n(t) = i \int_{\alpha_1}^{\alpha_2} \left\langle \psi_n(t') \left| \frac{\partial}{\partial \alpha} \psi_n(\alpha) \right. \right\rangle d\alpha \quad (7)$$

$$\frac{\partial}{\partial \alpha} \psi_n(\alpha) = e^{-m\alpha|x|/\hbar^2} \frac{m(2m\alpha|x| - \hbar^2)}{2\hbar^3 \sqrt{m\alpha}} \quad (8)$$

$$\left\langle \psi_n(t') \left| \frac{\partial}{\partial \alpha} \psi_n(\alpha) \right. \right\rangle = \frac{m}{2\hbar^4} \int_{-\infty}^{\infty} e^{-2m\alpha|x|/\hbar^2} (2m\alpha|x| - \hbar^2) dx \quad (9)$$

$$= \frac{m}{\hbar^4} \int_0^{\infty} e^{-2m\alpha x/\hbar^2} (2m\alpha x - \hbar^2) dx \quad (10)$$

$$= \frac{m}{2\hbar^2} x e^{-2m\alpha x/\hbar^2} \Big|_0^{\infty} \quad (11)$$

$$= 0 \quad (12)$$

Therefore  $\gamma_n = 0$ .

The bound state energy is

$$E = -\frac{m\alpha^2}{2\hbar^2} \quad (13)$$

so the dynamic phase is

$$\theta_n(t) = \frac{m}{2\hbar^3} \int_0^t \alpha^2(t') dt' \quad (14)$$

If  $\alpha$  changes at a constant rate, then  $d\alpha/dt = c$  and  $\alpha(t) = \alpha_1 + ct$ , so

$$\theta_n(t) = \frac{m}{2\hbar^3} \int_0^{(\alpha_2 - \alpha_1)/c} (\alpha_1 + ct)^2 dt \quad (15)$$

$$= \frac{m(\alpha_2^3 - \alpha_1^3)}{6c\hbar^3} \quad (16)$$

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