PHASES IN THE ADIABATIC THEOREM: DELTA FUNCTION WELL

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 10.4.

Here's another example of calculating phases in the adiabatic theorem which says that if a system starts out in the nth state of a time-dependent hamiltonian, and the hamiltonian changes slowly compared to the internal period of the time-independent wave function (that is, the time scale over which the hamiltonian changes is much longer than $\hbar/E_n$), then after a time t the system will end up in state

$$\Psi_n(t) = e^{i\theta_n(t)} e^{i\gamma_n(t)} \psi_n(t) \tag{1}$$

where

$$\theta_n(t) \equiv -\frac{1}{\hbar}\int_0^t E_n(t')\,dt' \tag{2}$$

$$\gamma_n(t) \equiv i\int_0^t \left\langle \psi_n(t') \middle| \frac{\partial}{\partial t'}\psi_n(t') \right\rangle dt' \tag{3}$$

θ is called the *dynamic phase* and γ is called the *geometric phase*.

The wave functions $\psi_n(t)$ are the solutions of the eigenvalue equation at a particular time t:

$$H(t)\,\psi_n(t) = E_n(t)\,\psi_n(t) \tag{4}$$

That is, they aren't a full solution of the time dependent Schrödinger equation; rather they are the solutions of the time-independent Schrödinger equation with whatever parameters are now time-dependent in the hamiltonian replaced by their time-dependent forms.

With a delta function well the potential is

$$V(x) = -\alpha\delta(x) \tag{5}$$

and the time-independent wave function for the bound state is

$$\psi(x)=\frac{\sqrt{m\alpha}}{\hbar}e^{-m\alpha|x|/\hbar^2} \tag{6}$$

If the strength of the delta function α varies with time, then

$$\gamma_n(t)=i\int_{\alpha_1}^{\alpha_2}\left\langle\psi_n(t')\left|\frac{\partial}{\partial\alpha}\psi_n(\alpha)\right.\right\rangle d\alpha \tag{7}$$

$$\frac{\partial}{\partial\alpha}\psi_n(\alpha)=e^{-m\alpha|x|/\hbar^2}\frac{m\left(2m\alpha|x|-\hbar^2\right)}{2\hbar^3\sqrt{m\alpha}} \tag{8}$$

$$\left\langle\psi_n(t')\left|\frac{\partial}{\partial\alpha}\psi_n(\alpha)\right.\right\rangle=\frac{m}{2\hbar^4}\int_{-\infty}^{\infty}e^{-2m\alpha|x|/\hbar^2}\left(2m\alpha|x|-\hbar^2\right)dx \tag{9}$$

$$=\frac{m}{\hbar^4}\int_0^{\infty}e^{-2m\alpha x/\hbar^2}\left(2m\alpha x-\hbar^2\right)dx \tag{10}$$

$$=\frac{m}{2\hbar^2}xe^{-2m\alpha x/\hbar^2}\Big|_0^{\infty} \tag{11}$$

$$=0 \tag{12}$$

Therefore $\gamma_n=0$.

The bound state energy is

$$E=-\frac{m\alpha^2}{2\hbar^2} \tag{13}$$

so the dynamic phase is

$$\theta_n(t)=\frac{m}{2\hbar^3}\int_0^t\alpha^2(t')\,dt' \tag{14}$$

If α changes at a constant rate, then $d\alpha/dt=c$ and $\alpha(t)=\alpha_1+ct$, so

$$\theta_n(t)=\frac{m}{2\hbar^3}\int_0^{(\alpha_2-\alpha_1)/c}(\alpha_1+ct)^2\,dt \tag{15}$$

$$=\frac{m\left(\alpha_2^3-\alpha_1^3\right)}{6c\hbar^3} \tag{16}$$

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