

INFINITE SQUARE WELL WITH VARIABLE DELTA FUNCTION BARRIER: LOCATION OF THE PARTICLE

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 10.8.

Continuing our example of the infinite square well with the growing delta function barrier, we can now find the probability of the particle being found in the right section of the well. The wave function is

$$(0.1) \quad \psi(x) = \begin{cases} Ae^{ikx} + Be^{-ikx} & 0 \leq x < \frac{a}{2} + \epsilon \\ Ce^{ikx} + De^{-ikx} & \frac{a}{2} + \epsilon < x \leq a \end{cases}$$

With the boundary conditions we can solve for 3 of the coefficients in terms of the fourth and we got

$$(0.2) \quad B = -A$$

$$(0.3) \quad A = e^{-iz} D \frac{\sin \left[\frac{1}{2}z(1 - \delta) \right]}{\sin \left[\frac{1}{2}z(1 + \delta) \right]}$$

$$(0.4) \quad C = -De^{-2iz}$$

where $z \equiv ka$.

We can call the left and right wave functions ψ_l and ψ_r and we get

$$(0.5) \quad \psi_l = 2ie^{-iz} D \frac{\sin \left[\frac{1}{2}z(1 - \delta) \right]}{\sin \left[\frac{1}{2}z(1 + \delta) \right]} \sin kx$$

$$(0.6) \quad \psi_r = 2ie^{-iz} D \sin(z - kx)$$

The probability of being in the right well is

$$(0.7) \quad P_r = \frac{\int_{\frac{a}{2} + \epsilon}^a |\psi_r|^2 dx}{\int_0^{\frac{a}{2} + \epsilon} |\psi_l|^2 dx + \int_{\frac{a}{2} + \epsilon}^a |\psi_r|^2 dx}$$

With $\delta \equiv 2\epsilon/a$ this is

$$(0.8) \quad P_r = \frac{\int_{\frac{a}{2}(1+\delta)}^a |\psi_r|^2 dx}{\int_0^{\frac{a}{2}(1+\delta)} |\psi_l|^2 dx + \int_{\frac{a}{2}(1+\delta)}^a |\psi_r|^2 dx}$$

We get

$$(0.9) \quad X_- \equiv \int_{\frac{a}{2}(1+\delta)}^a |\psi_r|^2 dx = \frac{D^2}{4k} [z(1-\delta) - \sin(z(1-\delta))]$$

$$(0.10) \quad X_+ \equiv \int_0^{\frac{a}{2}(1+\delta)} |\psi_l|^2 dx = \frac{D^2 \sin^2 \left[\frac{1}{2}z(1-\delta) \right]}{4k \sin^2 \left[\frac{1}{2}z(1+\delta) \right]} \left[[z(1+\delta) - 2 \sin \left[\frac{z}{2}(1+\delta) \right] \cos \left[\frac{z}{2}(1+\delta) \right]] \right]$$

$$(0.11) \quad = \frac{D^2 \sin^2 \left[\frac{1}{2}z(1-\delta) \right]}{4k \sin^2 \left[\frac{1}{2}z(1+\delta) \right]} (z(1+\delta) - \sin(z(1+\delta)))$$

So

$$(0.12) \quad P_r = \frac{X_-}{X_+ + X_-}$$

$$(0.13) \quad = \frac{1}{1 + X_+/X_-}$$

with

$$(0.14) \quad \frac{X_+}{X_-} = \frac{z(1+\delta) - \sin(z(1+\delta))}{z(1-\delta) - \sin(z(1-\delta))} \times \frac{\sin^2 \left[\frac{1}{2}z(1-\delta) \right]}{\sin^2 \left[\frac{1}{2}z(1+\delta) \right]} \equiv I_+$$

$$(0.15) \quad I_+ \equiv [z(1+\delta) - \sin(z(1+\delta))] \sin^2 \left[\frac{1}{2}z(1-\delta) \right]$$

$$(0.16) \quad I_- \equiv [z(1-\delta) - \sin(z(1-\delta))] \sin^2 \left[\frac{1}{2}z(1+\delta) \right]$$

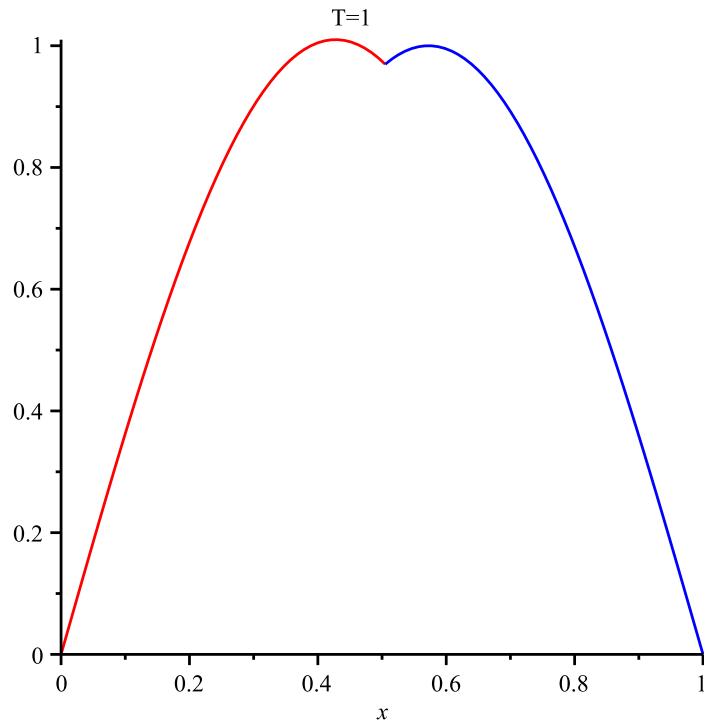
We can work out this probability for several values of T , with $\delta = 0.01$:

T	z	P_r
1	3.673	0.487
5	4.760	0.471
20	5.720	0.401
100	6.135	0.147
1000	6.215	0.00248

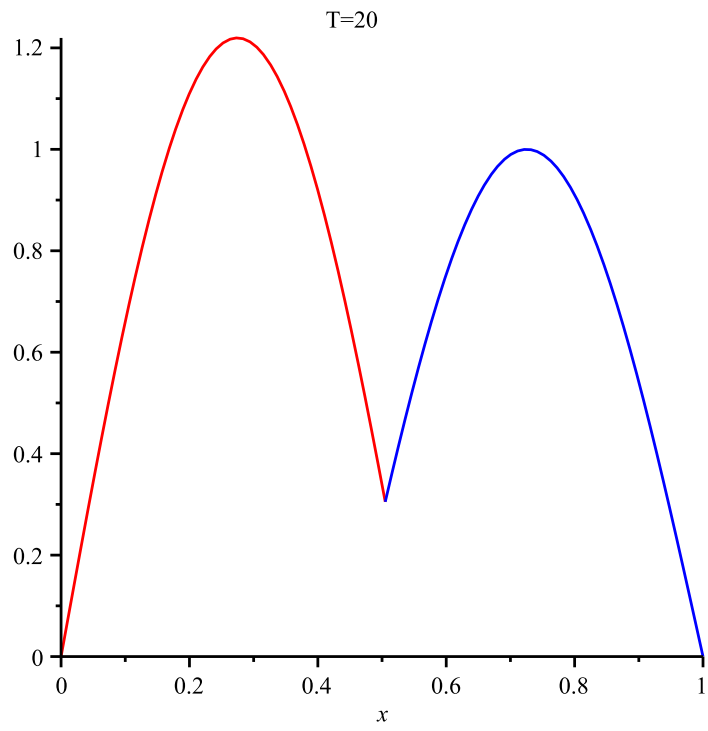
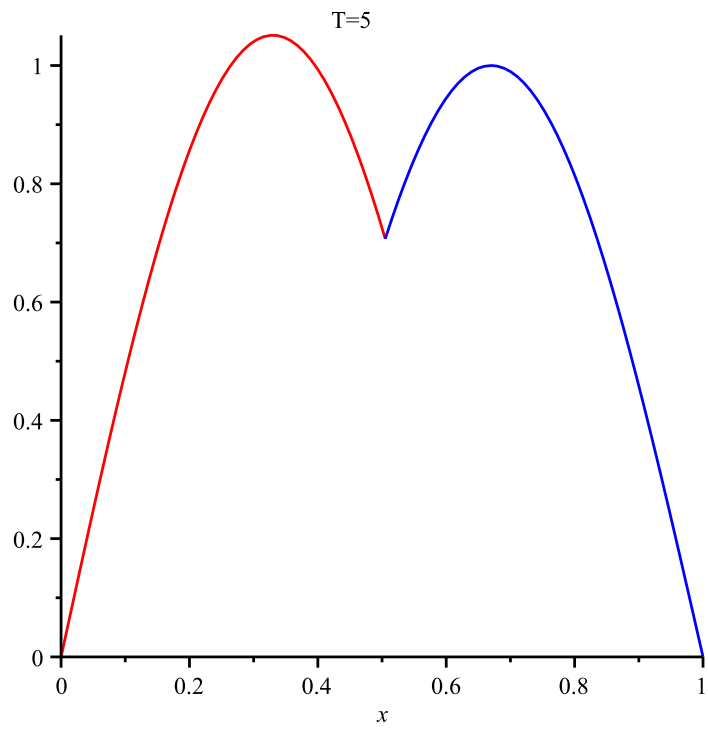
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The probability of the particle being to the right of the barrier drops to zero as the barrier strength becomes infinite, which is just what we saw above. Because the left well has a lower ground state energy, the particle favours that region.

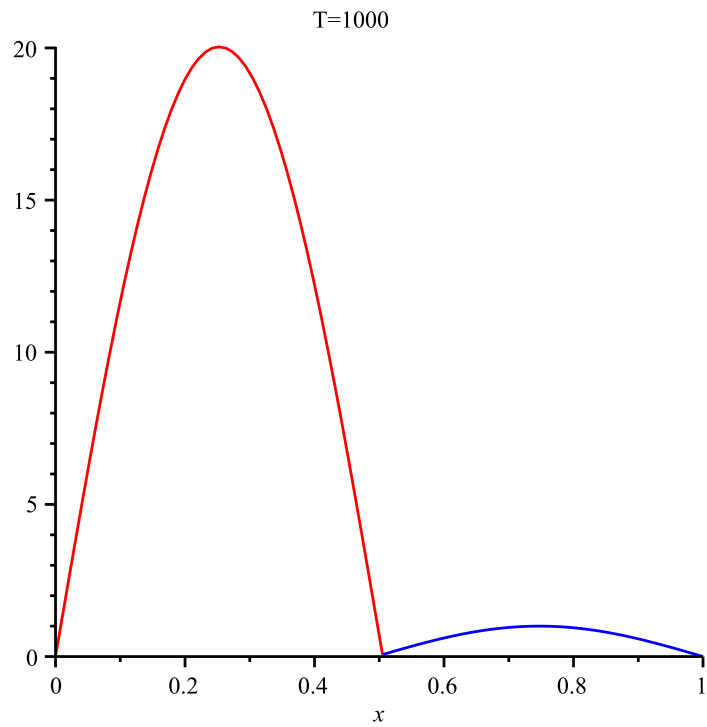
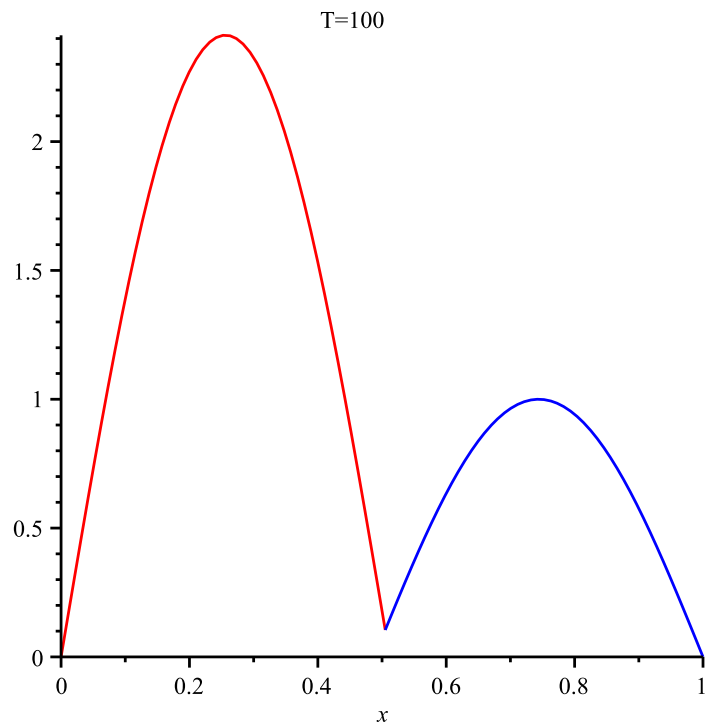
We can see this by plotting the wave functions 0.5 and 0.6 for these same values of T . We've used $\delta = 0.01$, $a = 1$ and omitted the common factor of $2ie^{-iz}D$ in the plots, so the wave functions aren't normalized, but all we're interested in is the relative wave functions in the two regions of the well.



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We can see that the wave function drops off as T increases.