BORN APPROXIMATION FOR A SPHERICAL DELTA FUNCTION SHELL

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 11.13.

Earlier we looked at scattering from a delta function spherical shell for a low energy incident particle, using partial wave analysis. This was a fairly complex task, as it involved matching interior and exterior wave functions at the delta function boundary.

Here, we'll calculate the scattering amplitude using the first Born approximation. For a spherically symmetric potential, the approximation is

$$f(\theta)\approx-\frac{2m}{\hbar^{2}\kappa}\int_{0}^{\infty}V(r)\,r\sin(\kappa r)\,dr \tag{1}$$

where

$$\kappa\equiv2k\sin\frac{\theta}{2} \tag{2}$$

For a delta function potential

$$V(r)=\alpha\delta(r-a) \tag{3}$$

where α is a constant representing the strength of the delta function, so we get

$$f(\theta)\approx-\frac{2m\alpha a}{\hbar^{2}\kappa}\sin(\kappa a) \tag{4}$$

For low energy, $ka\ll1$ so $\kappa a\ll1$ as well, so $\sin(\kappa a)\approx\kappa a$, and we get

$$f(\theta)\approx-\frac{2m\alpha a^{2}}{\hbar^{2}} \tag{5}$$

Our earlier result using partial wave analysis is

$$f(\theta)\approx-\frac{\beta a}{1+\beta} \tag{6}$$

where

$$\beta\equiv\frac{2m\alpha a}{\hbar^{2}} \tag{7}$$

This gives a differential cross section and total cross section of

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 = \beta^2 a^2 \tag{8}$$

$$\sigma = 4\pi\beta^2 a^2 \tag{9}$$

The low energy result 5 from the Born approximation is, in terms of β:

$$f(\theta) \approx -\beta a \tag{10}$$

so it agrees with the partial wave result if $\beta \ll 1$. This is equivalent to the condition that $\alpha \ll \hbar^2/2ma$, in other words, that the potential is weak. This was the main assumption in deriving the Born approximation, so in this limit, the results are consistent.