

BORN APPROXIMATION FOR A SPHERICAL DELTA FUNCTION SHELL

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 11.13.

Earlier we looked at scattering from a delta function spherical shell for a low energy incident particle, using partial wave analysis. This was a fairly complex task, as it involved matching interior and exterior wave functions at the delta function boundary.

Here, we'll calculate the scattering amplitude using the first Born approximation. For a spherically symmetric potential, the approximation is

$$(1) \quad f(\theta) \approx -\frac{2m}{\hbar^2 \kappa} \int_0^\infty V(r) r \sin(\kappa r) dr$$

where

$$(2) \quad \kappa \equiv 2k \sin \frac{\theta}{2}$$

For a delta function potential

$$(3) \quad V(r) = \alpha \delta(r - a)$$

where α is a constant representing the strength of the delta function, so we get

$$(4) \quad f(\theta) \approx -\frac{2m\alpha a}{\hbar^2 \kappa} \sin(\kappa a)$$

For low energy, $ka \ll 1$ so $\kappa a \ll 1$ as well, so $\sin(\kappa a) \approx \kappa a$, and we get

$$(5) \quad f(\theta) \approx -\frac{2m\alpha a^2}{\hbar^2}$$

Our earlier result using partial wave analysis is

$$(6) \quad f(\theta) \approx -\frac{\beta a}{1 + \beta}$$

where

$$(7) \quad \beta \equiv \frac{2m\alpha a}{\hbar^2}$$

This gives a differential cross section and total cross section of

$$(8) \quad \frac{d\sigma}{d\Omega} = |f(\theta)|^2 = \beta^2 a^2$$

$$(9) \quad \sigma = 4\pi\beta^2 a^2$$

The low energy result 5 from the Born approximation is, in terms of β :

$$(10) \quad f(\theta) \approx -\beta a$$

so it agrees with the partial wave result if $\beta \ll 1$. This is equivalent to the condition that $\alpha \ll \hbar^2/2ma$, in other words, that the potential is weak. This was the main assumption in deriving the Born approximation, so in this limit, the results are consistent.