

BORN APPROXIMATION IN ONE DIMENSION

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 11.17.

Using Griffiths's Green's function for the one-dimensional Schrödinger equation:

$$\psi(x) = \psi_0(x) - \frac{im}{\hbar^2 k} \int_{-\infty}^{\infty} e^{ik|x-x_0|} V(x_0) \psi(x_0) dx_0 \quad (1)$$

we can work out the Born approximation in the one-dimensional case. The idea is we replace $\psi(x_0)$ inside the integral by the incident plane wave form $\psi_0(x_0)$. Assuming that the potential is zero outside a finite distance from the origin, we want the wave function in two regions: $x \ll 0$ and $x \gg 0$. The former will give the reflected wave and the latter the transmitted wave.

For $x \ll 0$ (and at a distance where $V(x) = 0$) we have

$$e^{ik|x-x_0|} = e^{-ikx} e^{ikx_0} \quad (2)$$

$$\psi_0(x_0) = A e^{ikx_0} \quad (3)$$

where A is the normalization constant. The Born approximation for this region is

$$\psi(x) = A e^{ikx} - \frac{im}{\hbar^2 k} e^{-ikx} \int_{-\infty}^{\infty} e^{ikx_0} V(x_0) A e^{ikx_0} dx_0 \quad (4)$$

$$= A \left[e^{ikx} - \frac{im}{\hbar^2 k} e^{-ikx} \int_{-\infty}^{\infty} e^{2ikx_0} V(x_0) dx_0 \right] \quad (5)$$

The scattering amplitude and reflection coefficient for the reflected particle are therefore

$$f_R = -\frac{im}{\hbar^2 k} \int_{-\infty}^{\infty} e^{2ikx_0} V(x_0) dx_0 \quad (6)$$

$$R = |f_R|^2 \quad (7)$$

$$= \left(\frac{m}{\hbar^2 k} \right)^2 \left| \int_{-\infty}^{\infty} e^{2ikx_0} V(x_0) dx_0 \right|^2 \quad (8)$$

[Note that we're assuming that $x < x_0$ so although the limits of the integral are infinite, we're implicitly assuming that $V = 0$ for all $x_0 > x$ so the integral isn't really over an infinite range.]

For $x \gg 0$ we have

$$e^{ik|x-x_0|} = e^{ikx} e^{-ikx_0} \quad (9)$$

$$\psi_0(x_0) = Ae^{ikx_0} \quad (10)$$

The Born approximation here is

$$\psi(x) = Ae^{ikx} - \frac{im}{\hbar^2 k} e^{ikx} \int_{-\infty}^{\infty} e^{-ikx_0} V(x_0) Ae^{ikx_0} dx_0 \quad (11)$$

$$= Ae^{ikx} \left[1 - \frac{im}{\hbar^2 k} \int_{-\infty}^{\infty} V(x_0) dx_0 \right] \quad (12)$$

The scattering amplitude and transmission coefficient are therefore

$$f_T = 1 - \frac{im}{\hbar^2 k} \int_{-\infty}^{\infty} V(x_0) dx_0 \quad (13)$$

$$T = |f_T|^2 \quad (14)$$

$$= 1 + \left(\frac{m}{\hbar^2 k} \right)^2 \left| \int_{-\infty}^{\infty} V(x_0) dx_0 \right|^2 \quad (15)$$

The Born approximation fails for transmission in this case, since $T > 1$ which is impossible. We can still get an estimate of T from $T = 1 - R$.

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