BORN APPROXIMATION IN ONE DIMENSION

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 11.17.

Using Griffiths's Green's function for the one-dimensional Schrödinger equation:

$$\psi(x)=\psi_0(x)-\frac{im}{\hbar^2 k}\int_{-\infty}^{\infty}e^{ik|x-x_0|}V(x_0)\psi(x_0)\,dx_0 \tag{1}$$

we can work out the Born approximation in the one-dimensional case. The idea is we replace $\psi(x_0)$ inside the integral by the incident plane wave form $\psi_0(x_0)$. Assuming that the potential is zero outside a finite distance from the origin, we want the wave function in two regions: $x \ll 0$ and $x \gg 0$. The former will give the reflected wave and the latter the transmitted wave.

For $x \ll 0$ (and at a distance where $V(x)=0$) we have

$$\begin{aligned} e^{ik|x-x_0|} &= e^{-ikx}e^{ikx_0} && (2)\\ \psi_0(x_0) &= Ae^{ikx_0} && (3) \end{aligned}$$

where A is the normalization constant. The Born approximation for this region is

$$\begin{aligned} \psi(x) &= Ae^{ikx}-\frac{im}{\hbar^2 k}e^{-ikx}\int_{-\infty}^{\infty}e^{ikx_0}V(x_0)Ae^{ikx_0}dx_0 && (4)\\ &= A\left[e^{ikx}-\frac{im}{\hbar^2 k}e^{-ikx}\int_{-\infty}^{\infty}e^{2ikx_0}V(x_0)\,dx_0\right] && (5) \end{aligned}$$

The scattering amplitude and reflection coefficient for the reflected particle are therefore

$$f_R = -\frac{im}{\hbar^2 k}\int_{-\infty}^{\infty} e^{2ikx_0} V(x_0)\,dx_0 \tag{6}$$

$$R = |f_R|^2 \tag{7}$$

$$= \left(\frac{m}{\hbar^2 k}\right)^2 \left|\int_{-\infty}^{\infty} e^{2ikx_0} V(x_0)\,dx_0\right|^2 \tag{8}$$

[Note that we're assuming that $x < x_0$ so although the limits of the integral are infinite, we're implicitly assuming that $V = 0$ for all $x_0 > x$ so the integral isn't really over an infinite range.]

For $x \gg 0$ we have

$$e^{ik|x-x_0|} = e^{ikx}e^{-ikx_0} \tag{9}$$

$$\psi_0(x_0) = Ae^{ikx_0} \tag{10}$$

The Born approximation here is

$$\psi(x) = Ae^{ikx} - \frac{im}{\hbar^2 k}e^{ikx}\int_{-\infty}^{\infty} e^{-ikx_0} V(x_0)\,Ae^{ikx_0}dx_0 \tag{11}$$

$$= Ae^{ikx}\left[1 - \frac{im}{\hbar^2 k}\int_{-\infty}^{\infty} V(x_0)\,dx_0\right] \tag{12}$$

The scattering amplitude and transmission coefficient are therefore

$$f_T = 1 - \frac{im}{\hbar^2 k}\int_{-\infty}^{\infty} V(x_0)\,dx_0 \tag{13}$$

$$T = |f_T|^2 \tag{14}$$

$$= 1 + \left(\frac{m}{\hbar^2 k}\right)^2 \left|\int_{-\infty}^{\infty} V(x_0)\,dx_0\right|^2 \tag{15}$$

The Born approximation fails for transmission in this case, since $T > 1$ which is impossible. We can still get an estimate of T from $T = 1 - R$.

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