

BORN APPROXIMATION OF DELTA FUNCTION WELL AND FINITE SQUARE WELL

Link to: [physicspages home page](#).

To leave a comment or report an error, please use the [auxiliary blog](#).

Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 11.18.

We can use the one-dimensional Born approximation to get approximate values for transmission through a couple of test potentials. The Born approximation gives a reflection coefficient of

$$R = \left(\frac{m}{\hbar^2 k} \right)^2 \left| \int_{-\infty}^{\infty} e^{2ikx} V(x) dx \right|^2 \quad (1)$$

with

$$k = \frac{\sqrt{2mE}}{\hbar} \quad (2)$$

Example 1. The delta function well. Here

$$V = -\alpha\delta(x) \quad (3)$$

so

$$R = \left(\frac{m}{\hbar^2 k} \right)^2 \left| -\alpha e^{2ik \times 0} \right|^2 \quad (4)$$

$$= \frac{m\alpha^2}{2\hbar^2 E} \quad (5)$$

$$T = 1 - R = 1 - \frac{m\alpha^2}{2\hbar^2 E} \quad (6)$$

The exact values are

$$R = \frac{1}{1 + 2\hbar^2 E / m\alpha^2} \quad (7)$$

$$T = \frac{1}{1 + m\alpha^2 / 2\hbar^2 E} \quad (8)$$

For a weak potential, α is small, so $2\hbar^2 E / m\alpha^2 \gg 1$ so we can approximate the exact formulas by

$$R \approx \frac{1}{2\hbar^2 E/m\alpha^2} = \frac{m\alpha^2}{2\hbar^2 E} \quad (9)$$

$$T \approx 1 - \frac{m\alpha^2}{2\hbar^2 E} \quad (10)$$

in agreement with the Born approximation.

Example 2. The finite square well. Here

$$V = \begin{cases} -V_0 & -a < x < a \\ 0 & \text{otherwise} \end{cases} \quad (11)$$

We get from 1

$$\int_{-\infty}^{\infty} e^{2ikx} V(x) dx = -V_0 \int_{-a}^a e^{2ikx} dx \quad (12)$$

$$= -V_0 \frac{e^{2ika} - e^{-2ika}}{2ik} \quad (13)$$

$$= -\frac{V_0}{k} \sin(2ka) \quad (14)$$

$$R = \left(\frac{m}{\hbar^2 k} \right)^2 \left| -\frac{V_0}{k} \sin(2ka) \right|^2 \quad (15)$$

$$= \frac{V_0^2}{4E^2} \sin^2 \left(\frac{2a\sqrt{2mE}}{\hbar} \right) \quad (16)$$

$$T = 1 - \frac{V_0^2}{4E^2} \sin^2 \left(\frac{2a\sqrt{2mE}}{\hbar} \right) \quad (17)$$

The exact formula is

$$T^{-1} = 1 + \frac{V_0^2}{4E(E+V_0)} \sin^2 \left(\frac{2a}{\hbar} \sqrt{2m(E+V_0)} \right) \quad (18)$$

For a weak potential (or, equivalently, a high incident energy), $E \gg V_0$ and we get

$$T^{-1} \approx 1 + \frac{V_0^2}{4E^2} \sin^2 \left(\frac{2a\sqrt{2mE}}{\hbar} \right) \quad (19)$$

$$T \approx 1 - \frac{V_0^2}{4E^2} \sin^2 \left(\frac{2a\sqrt{2mE}}{\hbar} \right) \quad (20)$$

BORN APPROXIMATION OF DELTA FUNCTION WELL AND FINITE SQUARE WELL 3

so again, the Born approximation gives a good result.