OPTICAL THEOREM

There is a simple relationship between the total cross section and the scattering amplitude in three dimensional scattering. The formulas are (in terms of phase shifts):

\[ f(\theta) = \frac{1}{k} \sum_{l=0}^{\infty} (2l + 1) e^{i\delta_l} \sin \delta_l P_l(\cos \theta) \]  \hspace{1cm} (1)

\[ \sigma = \frac{4\pi}{k^2} \sum_{l=0}^{\infty} (2l + 1) \sin^2 \delta_l \]  \hspace{1cm} (2)

If \( \theta = 0 \), we can use the fact that \( P_l(1) = 1 \) for all \( l \) (from the definition) so taking the imaginary part of \( f(0) \) we get

\[ \Im f(0) = \frac{1}{k} \sum_{l=0}^{\infty} (2l + 1) \sin^2 \delta_l \]  \hspace{1cm} (3)

from which we get the optical theorem:

\[ \sigma = \frac{4\pi}{k} \Im f(0) \]  \hspace{1cm} (4)