KINETIC ENERGY

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The concept of energy is central to physics. Indeed, it could be said that, since Einstein showed that mass and energy are equivalent through the relation $E = mc^2$, physics is the study of energy and little else. However, it's a difficult quantity to define. What exactly *is* energy? The standard definition is that it is "the capacity to do work". In that sense it is broadly the same definition as in everyday language. Someone with a lot of energy is able to do a lot of work or undertake a lot of activity.

This definition, however, merely transfers the problem from that of defining energy to that of defining work. In physics, 'work' has a precise definition, which is that it is the energy produced by the action of a force over a certain distance. We can use this definition together with Newton's laws to get some idea of how to measure the energy in a system of objects.

Newton's laws are encapsulated in what may be the second most famous equation in physics: $F = ma$, which states that a force F applied to a mass m results in an acceleration a. Note that this relation is an *assumption*; that is, it is a statement of how Newton viewed the world, and is derived from observations, not from any prior theory.

To apply Newton's law to determine the energy produced by a force acting on a mass, we can look at a simple system where a constant force acts on a constant mass over some distance s. Using the definition of work above, the amount of work done by the force is therefore Fs. If we assume further that this force is unopposed, the entire effect of the force is to accelerate the mass. How can we relate the acceleration to the distance over which the mass moves?

Acceleration is the rate of change of velocity, and velocity is the rate of change of position. For this simple case, we'll make the further assumptions that (a) the mass starts at rest and with zero acceleration and (b) that the motion is in one dimension (along a straight line). This means that we can treat the acceleration, velocity and position as scalar quantities (that is, quantities that can be specified by giving a numerical value only, rather than both a value and a direction) rather than vectors, which simplifies things a fair bit. We'll deal with vectors in another post.

The easiest way to get the relation between acceleration, velocity and position is to use calculus, but to ease the calculus-shy reader into the subject gently, we can treat the simple case described above purely by using algebra.

Since the mass is accelerating, its velocity v is not constant, but since the *acceleration* is constant, and acceleration is the rate of change of velocity, the *rate* at which v is changing is constant and is equal to the acceleration a. Thus after a certain time t the velocity will be $v = at$. Since the velocity is changing at a constant rate, the *average* velocity $\bar{v}$ over the time t is just half the final velocity, so $\bar{v} = at/2$. Now the distance moved in time t will be the average velocity times the time, so $s = \bar{v}t = at^2/2$. (As we'll see below, when we do use calculus, this result is also true in general.)

So where does this get us? Well, we can now get an expression for the work done by the force in terms of the final velocity attained by the mass. Since the work done by the force is just the energy imparted to the mass, we will use the letter E to stand for this work. Then we get

$$E = Fs = (ma)(at^2/2) = m(at)^2/2 = mv^2/2 \tag{1}$$

In the last step we used the fact that the final velocity is $v = at$.

For anyone who has studied physics before, the final expression on the right will be recognized as the familiar expression for *kinetic energy*. The kinetic energy of a mass is the energy due to its motion, so it depends on the velocity as we can see here.

In the real world, of course, it is virtually impossible to convert all the work done by a force into motion, since most masses to which a force can be applied have internal structure which will be affected by the force, and will interact with other objects around them resulting in the energy being transmitted to these other objects by friction and other effects.

It is interesting to note that the kinetic energy depends on the *square* of the velocity, so if you double a mass's speed, its kinetic energy (its ability to do work, remember) increases by a factor of 4. This effect explains why, for example, the damage resulting from a car crash is so much more serious at higher speeds. A car travelling twice as fast has four times as much kinetic energy, so when the car hits something, the work done on the hit object is four times as much.

Now as promised, here's a simple derivation of the kinetic energy formula using calculus.

The acceleration is the rate of change of velocity, so can be written as

$$a = \frac{dv}{dt} \tag{2}$$

Similarly, velocity is the rate of change of distance, so

$$v=\frac{ds}{dt} \tag{3}$$

If a is constant, then after a time t, the velocity will be

$$v=\int_0^t a\,dt'=at \tag{4}$$

and the position will be

$$s=\int_0^t v\,dt'=\int_0^t at'\,dt'=\frac{1}{2}at^2 \tag{5}$$

From here, the derivation of the kinetic energy is the same as that given above.

As an exercise, you might like to work out the more general case of the position of a mass that starts at some non-zero location $s(t=0)=s_0$ with a non-zero initial velocity $v(t=0)=v_0$. You should find that the position as a function of time $s(t)$ is

$$s(t)=s_0+v_0t+\frac{1}{2}at^2 \tag{6}$$

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