POSITION OPERATOR - EIGENFUNCTIONS

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education, sec 3.3.

We've seen that we can calculate the eigenfunctions of the momentum operator by solving a simple differential equation. Finding the eigenfunctions of the position operator requires a different approach since no differential equation is involved. Its eigenvalue equation is

$$x \left| e_{x'} \right\rangle = x' \left| e_{x'} \right\rangle \tag{1}$$

where the x on the LHS is the position *operator*, even though it just multiplies its eigenfunction, and the x' on the RHS is a *constant* eigenvalue. Thus we're looking for an eigenfunction which has the property that when it's multiplied by x it gives back the same function multiplied by a constant x'.

At first glance, you might think that we can just set $x = x'$ and this equation is then true for any function. However, remember that we're trying to find a function such that multiplying it by the continuous variable x results in a *fixed* value x' multiplied by the same function. If this confuses you, think back to the hamiltonian and its eigenvalue equation $H\psi_n = E_n\psi_n$. Here we have a single operator H which, when operating on a particular function ψ_n gives back that same function multiplied by a constant E_n. If we give it a different eigenfunction, the same operator will multiply this eigenfunction by a different eigenvalue.

In the case of the position operator, then, we want to find a function that, when operated on by the operator x gives back the same function multiplied by a particular value x'. This function can be zero everywhere except at $x = x'$, which leads us to using the delta function as the eigenfunction. This is because

$$x\delta(x - x') = x'\delta(x - x') \tag{2}$$

where, remember, the x is a continuous variable and the x' is a constant. The delta function picks out the one value of x where $x = x'$.

Thus we can say

$$|e_{x'}\rangle = A\delta(x - x') \tag{3}$$

for some constant A.

This function isn't square integrable, but if we try to calculate the inner product we get

$$\langle e_z \,|e_y\rangle = |A|^2 \int \delta(x - z)\delta(x - y)dx \tag{4}$$

$$= |A|^2 \delta(z - y) \tag{5}$$

If we pick $A = 1$, we get a sort of pseudo-orthonormality condition:

$$\langle e_z \,|e_y\rangle = \delta(z - y) \tag{6}$$

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