POTENTIAL VERSUS POTENTIAL ENERGY

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A common source of confusion is the difference between the terms *potential energy* and just *potential*. Both terms refer to forces that are *conservative*, so we'd best start with a definition of that term.

Suppose you place a mass at some point in space where a force is acting (it doesn't really matter which force, but if you want to visualize one for the purposes of the argument, gravity is as good as any). Now if you move the mass on some path through space and eventually end up back where you started, how much work is done? For a conservative force, the answer is: no work at all. Suppose the force is an attractive force (so it could be the gravitational force due to a star, say). If you are moving in a closed loop, then for every bit of the loop where you are moving against the force (that is, moving away from the star), there has to be some other part of the loop where you are moving through the same distance *with* the force (towards the star). Since the work done by the force when the mass is moving against it is equal and opposite to the work done by the force when the mass is moving with it, the bits of work done all cancel out in pairs and the net work done is zero.

You might think that in order to move against a force, you need to exert some sort of other, external force. For example, on the Earth's surface it is certainly true that it takes a lot of work to lift a mass against the pull of the Earth's gravity, and that work must come from your own muscles or from the expenditure of some other kind of energy if you are using a machine to do the lifting. In those situations, it is true that work is expended by other forces when going against gravity. But suppose you are looking at a planet orbiting a star. If the planet is in a non-circular (e.g. elliptical) orbit, then there will be some parts of the orbit where the planet is getting closer to the star (working with gravity) and other parts where it is moving away from the star (working against gravity). There is no external force involved; it is merely planet's inertia which carries it away from the star on that part of its orbit where the distance between it and the star increases. Since the planet's orbit will eventually take it back to its starting point (we're neglecting the effects of precession here), a closed loop has been traversed in a conservative force field, so no work has been done.

Lest you think that all forces are conservative (since the vast majority of those that are studied in physics courses are), it is possible to define forces that are not. The most common such force is friction: the work done by a friction force is always negative, since you are always moving against it; there is no way of moving a mass *with* a friction force and thus get friction to do some positive work. Thus if you placed a mass on a rough surface like sandpaper and moved it in some path which ended up where it started, the amount of work would not only not be zero, but it *would* depend on the path taken: a longer route would require more work than a shorter one. As such, it makes no sense to define potential energy when talking about friction; all the work that is done against friction is simply lost as heat and cannot be reclaimed as kinetic energy by somehow contriving to work 'with' friction.

The key point about a conservative force, then, is that it is possible to define a difference in energy between two locations by specifying only the locations in space; there is no need to specify how you got to those locations. So when you move from point A to point B in a gravitational field, the gravitational force does a fixed amount of work on you (which will be positive if you are moving with the field and negative if you are moving against it). If you are moving with the field (known in everyday language as 'falling') the field is doing work on you and you are accelerating, thus your kinetic energy is increasing. When you are moving against the field (as in climbing stairs or going up in a hot air balloon) you are doing work against the force, so the work done by the force is negative, and your kinetic energy is decreasing.

Since energy must be conserved in a closed system, these changes in kinetic energy must be balanced somehow, and that is where the notion of *potential energy* comes in. What is gained by kinetic energy is lost to potential energy and vice versa. Rather than having to worry about kinetic energy to calculate potential energy, however, it seems easier to define the change in potential energy as the difference between the values of some function which is defined at each point in space. This function is the *potential function*. Thus potential energy is the difference in potential between two locations in space.

Since the gain or loss of potential energy can also be calculated by finding the negative of the work done by the force, and the work is the integral of (force) times (distance), it is this latter function which is defined to be the potential function. That is, for a conservative force,

$$Force = -\frac{d(Potential)}{dx} \tag{1}$$

This formula is in one dimension; two or three dimensional analogues are easily defined using vector calculus, so the single derivative becomes a gradient.

So, for example, for simple harmonic motion, since $F=-kx$, the corresponding potential (usually given the symbol V) is

$$V = -\int F\,dx \tag{2}$$

$$= \frac{1}{2}kx^2 \tag{3}$$

For the gravitational field, $F=GMm/r^2$, so $V=-GMm/r$ and so on.

Any force which can be expressed as the gradient of a potential function is a conservative force.

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