

## THE SCHRÖDINGER EQUATION

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There are various ways to introduce quantum mechanics. Probably the most common is the historical approach, in which we see some of the problems that arose with classical physics, and how these problems led several physicists to propose novel approaches that ultimately gave rise to quantum theory. Another approach is to begin with the Schrödinger equation, which is, essentially, the quantum statement of the total energy of an object. Although it took close to 30 years from the time the idea of the quantum was first proposed until Schrödinger wrote down his equation, the Schrödinger equation itself (which we will call the SE from now on) can be considered as the fundamental postulate of quantum mechanics (at least, non-relativistic quantum mechanics).

One caution must be made for any reader who is venturing into quantum mechanics for the first time. The ideas behind quantum mechanics and many of its predictions are just plain weird. There's no getting around this, and the most eminent physicists who were involved in creating the theory, as well as equally eminent ones who have worked on the theory since then, all agree that it is essentially impossible to get a clear intuitive mental picture of just what is happening in the subatomic world. Objects at that scale behave in ways that are completely foreign to our everyday experience. Things are so weird, in fact, that some physicists question whether quantum mechanics is truly correct, or whether it is an incorrect theory that just happens to give the right results. After all, they point out, Newtonian physics gave the right results for more than 200 years until people found tiny cracks in its predictions. The same could very well be true of quantum mechanics; only time will tell.

This article will take the direct approach by stating the SE as the fundamental postulate of non-relativistic quantum theory, and then describing some of the basic features of the equation.

The Schrödinger equation for a single particle moving in one dimension is:

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V\Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

In this equation,  $\hbar$  is Planck's constant divided by  $2\pi$ ,  $m$  is the constant mass of the particle and  $i = \sqrt{-1}$  is the unit of imaginary numbers. The

function  $V = V(x, t)$  is the potential in which the particle moves, and in the general case can depend on both position and time. All these quantities are assumed to be known when the equation is written down.

The unknown function is therefore the mysterious  $\Psi = \Psi(x, t)$ . This is the famous *wave function* which is central to all of quantum theory. First, we can have a look at some of the properties of  $\Psi$  from a mathematical point of view. The presence of the imaginary unit  $i$  tells us that  $\Psi$  can be a *complex* function; that is, in general its values are complex numbers. This in turn tells us that  $\Psi$  itself cannot represent a physically measurable quantity, since everything in the 'real' world must be representable by 'real' numbers (OK, I realize that I'm taking liberties here, since the 'real' in 'real world' isn't an exact term, whereas the 'real' in 'real number' is, but in this case it can only help the reader to realize that 'real world' quantities must also be 'real numbers'.)

If  $\Psi$  is complex, how can we use the SE to make any predictions of physically measurable quantities? This problem was solved (if solved is the right word) by the physicist Max Born when he proposed that it is only the square modulus  $|\Psi|^2 = \Psi^* \Psi$ , where  $\Psi^*$  is the complex conjugate of  $\Psi$ , which can have physical meaning. The square modulus of a complex number is always real, so it *can* represent a measurable quantity, but *what* exactly is this quantity?

Born's novel proposal was based on the idea that for many types of potential,  $\Psi$  has wavelike properties (which was one of the main reasons the SE was proposed - as an explanation of the wave properties of matter). In particular, it can exhibit interference (the phenomenon where two waves can add if their peaks coincide, or cancel out if the peak of one wave coincides with the trough of another).

Now one property of a wave is that it doesn't make sense to ask precisely where it is. After all, waves are pretty much (by definition) spread out over some region of space. All we can do is say how high or low the wave is at a given point (as with a water wave, where the height of the water varies over the extent of the wave).

However, quantum mechanics must also try to provide a way of describing matter as particles under some conditions, so if all we could do was interpret  $\Psi$  as a wave, it wouldn't be an adequate description of matter. So Born's proposal is that  $|\Psi(x, t)|^2$  (the physically measurable square modulus, where we have made explicit the dependence on space and time) is the *probability* (technically, the probability density) of finding the particle at location  $x$  at time  $t$ . If a particle behaves in a wavelike fashion, then it will have peaks and troughs in its wave function, and we would expect it to be found more often near peak positions than near trough positions.

Thus the Born interpretation of quantum mechanics is that each particle is described by a (complex) wave function  $\Psi$  whose physical interpretation is that its square modulus  $|\Psi(x, t)|^2$  is the probability of finding the particle at location  $x$  at time  $t$ . Once this is accepted, there really isn't that much more physics to do to get a working theory. In fact, most of the problems in a first course in quantum mechanics are mathematical, since as you might imagine, attempting to solve the SE for various types of potential often leads to functions not often encountered in elementary mathematics.

The fundamental problem in quantum mechanics, then, can be viewed as solving the SE to determine the wave function, since once that is found, all the other physically meaningful quantities can be calculated from it. It is true that there are many alternative methods for finding some of these quantities, as well as some elegant mathematical tricks and shortcuts that make the job easier, but ultimately all the physics is contained in the wave equation, and the wave equation arises as a solution of the SE.

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