WHY PHYSICS NEEDS CALCULUS

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

Beyond a very elementary stage, it isn't possible to make much progress in physics unless you know something about calculus. This statement in itself is quite remarkable, given that calculus wasn't invented (or perhaps 'discovered' is more accurate) until Newton and Leibniz came along in the latter half of the 17th century. By this measure, all the physics that had been done up until that time wasn't particularly advanced, and indeed in many cases it was just plain wrong.

So what is so important about calculus in physics? Beyond a very elementary stage, physics is primarily about quantities that change in time and/or space, and calculus is the branch of mathematics that deals with these types of changes.

Calculus has two main forms: differential and integral. These two forms are opposites in the same sense that addition and subtraction are opposites: each is the inverse operation of the other.

To get an initial feel for how calculus might apply to physics, consider an object (such as a person, although the actual type of object doesn't really matter) that is able to move in one dimension only. If you picture the object as yourself, you can imagine you are able to walk, run or ride a bike along a straight path. You can assume that the path is long enough that you can go as far as you like in either direction.

Imagine that the path runs along the north-south line, and call the point where you start out $y=0$. As you'll discover as you study physics, the units of measurement are very important to keep straight, so we'll assume that you are measuring your position along the path in metres (m). Thus your starting point is$y=0$ m. To be clear, we'll call positions north of the point $y=0$ positive, and positions south of this point negative. Thus if you walk along the path to a point 10 m north of your starting position, you are now at $y=+10$ or just $y=10$ (since a number without a sign in front of it is always taken as positive). If you walk 10 metres south, you are then at $y=-10$ m.

That's about all we can say if we restrict ourselves to saying just where we are standing. In order to allow for moving around, we need to introduce the idea of a speed. Suppose we start at $y=0$ m at time $t=0$ s (we will measure time in seconds (s) unless otherwise stated) walk 10 m north and arrive at $y=10$ m at time $t=2$ s. We can say that our average speed is

5 m/s, but even if we walk at a reasonbly constant rate, this won't be our speed at every instant of time. We know this because we start at $y=0$ when we are standing still, so our initial speed is zero. Similarly when we arrive at $y=10$ we stop, so again our speed is zero. Thus we must speed up at the start and slow down at the end, so our speed is not constant over the entire 2 seconds.

We are then faced with the problem of figuring out what our speed is at each point in time as we walk over the 10 metres. This is where the calculus comes in. We know that we can figure out the average speed of travel between any two points by measuring how far you walk and divide by how long it takes you to do it. So if you want to know your average speed over the time it takes you to walk the first metre, suppose you measure the time this takes and it comes out to 0.25 s. Your average speed over this first second is then (1 m)/(0.25 s) = 4 m/s. The time is a bit lower than your average speed, since when you start out, it takes you some time to speed up to the constant speed with which you cover the distance in the middle.

Next, suppose you wanted to find your average speed between $y=0.5$ and $y=1$. Again, you would need to measure the time it takes to cover this distance, so suppose you do this and it comes out to 0.09 seconds. Your average speed over this interval is $(1.0-0.5)\div(0.09)\approx 5.56$ m/s. (We've used the 'approximately equal' sign $\approx$ here, since the exact answer is a repeating decimal. The problem of round-off error is significant when doing calculations in physics, but we must leave that problem until later.) This time, the speed is a bit higher than the overall average.

We can continue the process for smaller and smaller intervals, so we could, for example, find the average speed over the distance between $y=0.5$ and $y=0.6$, then between $y=0.5$ and $y=0.51$ and so on. However, we are still calculating only average speeds between two different time intervals. Is it possible (indeed, does it have any meaning) to calculate the exact speed at a single, precise location? That is, can we find the precise speed at the exact time you are passing the point $y=0.5$?

Clearly we can't do this by measuring distances and times, since we are asking for a precise value of the speed at one particular instant, where the instant of time has zero duration. To do it properly, we need the concept of a limit, so see the page on limits, where we discuss why limits are needed, and what they mean.

PINGBACKS

Pingback: Derivatives
Pingback: Limits