FEYNMAN PROPAGATOR: FROM COMMUTATOR TO INTEGRAL

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Reference: References: Robert D. Klauber, *Student Friendly Quantum Field Theory*, (Sandtrove Press, 2013) - Chapter 3, Section 3.13.2.

We've seen that the creation and annihilation of a virtual particle or antiparticle is described by the Feynman propagator, defined as

$$i\Delta_F(x-y) \equiv \left\langle 0 \left| T\left[\phi(x)\phi^{\dagger}(y)\right]\right| 0\right\rangle \tag{1}$$

where T is the time ordering operator, which places its arguments (the fields) in the correct order to create and annihilate the particle or antiparticle. We can express the propagator in terms of commutators with the results, first for a particle created at t_y and annihilated at $t_x > t_y$

$$i\Delta_F(x-y) = \left\langle 0 \left| \left[\phi^{+}(x), \phi^{\dagger -}(y)\right]\right| 0\right\rangle \tag{2}$$

and secondly, for an antiparticle created at t_x and annihilated at $t_y > t_x$:

$$i\Delta_F(x-y) = \left\langle 0 \left| \left[\phi^{\dagger +}(y), \phi^{-}(x)\right]\right| 0\right\rangle \tag{3}$$

The various field operators are defined as

$$\phi(x) = \int \frac{d^3k}{\sqrt{2(2\pi)^3\omega_{\mathbf{k}}}} a(\mathbf{k}) e^{-ikx} + \int \frac{d^3k}{\sqrt{2(2\pi)^3\omega_{\mathbf{k}}}} b^{\dagger}(\mathbf{k}) e^{ikx} \tag{4}$$

$$\equiv \phi^{+} + \phi^{-} \tag{5}$$

$$\phi^{\dagger}(x) = \int \frac{d^3k}{\sqrt{2(2\pi)^3\omega_{\mathbf{k}}}} b(\mathbf{k}) e^{-ikx} + \int \frac{d^3k}{\sqrt{2(2\pi)^3\omega_{\mathbf{k}}}} a^{\dagger}(\mathbf{k}) e^{ikx} \tag{6}$$

$$\equiv \phi^{\dagger +} + \phi^{\dagger -} \tag{7}$$

Looking at the particle propagator 2, we see that the commutator becomes (using the symbol $i\Delta^{+}(x-y)$):

$$i\Delta^{+}(x-y) \equiv \left[\phi^{+}(x),\phi^{\dagger -}(y)\right] \tag{8}$$

$$= \frac{1}{2(2\pi)^{3}}\int d^{3}\mathbf{k}\int d^{3}\mathbf{k}'\left[a(\mathbf{k}),a^{\dagger}\left(\mathbf{k}'\right)\right]\frac{e^{-ikx}e^{ik'y}}{\sqrt{\omega_{\mathbf{k}}\omega_{\mathbf{k}'}}} \tag{9}$$

The commutator in the integrand is

$$\left[a(\mathbf{k}),a^{\dagger}\left(\mathbf{k}'\right)\right] = \delta\left(\mathbf{k}-\mathbf{k}'\right) \tag{10}$$

so the integral over $\mathbf{k}'$ has the effect of setting $\mathbf{k}'=\mathbf{k}$ everywhere, with the result

$$i\Delta^{+}(x-y) = \frac{1}{2(2\pi)^{3}}\int d^{3}\mathbf{k}\frac{e^{-ik(x-y)}}{\omega_{\mathbf{k}}} \tag{11}$$

Following through the same steps for the antiparticle case, we start from 3. The only difference is that the signs of x and y are reversed (since $\left[b(\mathbf{k}),b^{\dagger}(\mathbf{k}')\right]=\delta(\mathbf{k}-\mathbf{k}')$ as well), so we get

$$i\Delta^{-}(x-y) = \frac{1}{2(2\pi)^{3}}\int d^{3}\mathbf{k}\frac{e^{ik(x-y)}}{\omega_{\mathbf{k}}} \tag{12}$$

Or, in condensed form

$$i\Delta^{\pm}(x-y) = \frac{1}{2(2\pi)^{3}}\int d^{3}\mathbf{k}\frac{e^{\mp ik(x-y)}}{\omega_{\mathbf{k}}} \tag{13}$$

The nice thing about this form is that it contains no operators, only numerical functions, so in principle we can evaluate the integral to give a function of x and y which comes outside the bracket in 1, with the result that (since $\langle 0|0\rangle = 1$):

$$i\Delta_{F}(x-y) = \frac{1}{2(2\pi)^{3}}\int d^{3}\mathbf{k}\frac{e^{\mp ik(x-y)}}{\omega_{\mathbf{k}}} \tag{14}$$

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