CREATION AND DESTRUCTION OPERATORS FOR A FREE SCALAR FIELD

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Reference: References: Robert D. Klauber, *Student Friendly Quantum Field Theory*, (Sandtrove Press, 2013) - Chapter 3, Problem 3.10.

The scalar, free-field Hamiltonian derived earlier is

$$H=\sum_{\mathbf{k}}\omega_{\mathbf{k}}\left[N_a(\mathbf{k})+\frac{1}{2}+N_b(\mathbf{k})+\frac{1}{2}\right] \tag{1}$$

where the operators are

$$N_a(\mathbf{k}) = a^{\dagger}(\mathbf{k})\,a(\mathbf{k}) \tag{2}$$

$$N_b(\mathbf{k}) = b^{\dagger}(\mathbf{k})\,b(\mathbf{k}) \tag{3}$$

These operators can be interpreted as number operators. That is, when they operate on a state $|n_{\mathbf{k}}\rangle$ containing $n_{\mathbf{k}}$ particles with energy $\omega_{\mathbf{k}}$, their eigenvalues are just $n_{\mathbf{k}}$:

$$N_a(\mathbf{k})|n_{\mathbf{k}}\rangle=n_{\mathbf{k}}|n_{\mathbf{k}}\rangle \tag{4}$$

With this interpretation, and the commutation relations for the a and b operators, we can derive the fact that $a^{\dagger}(\mathbf{k})$ is a *creation operator* for type a particles with energy $\omega_{\mathbf{k}}$, $a(\mathbf{k})$ is a *destruction operator* for the same type of particle, and $b^{\dagger}(\mathbf{k})$ and $b(\mathbf{k})$ are the analogous operators for type b particles (antiparticles).

Suppose we operate on the state $|n_{\mathbf{k}}\rangle$ with $a^{\dagger}(\mathbf{k})$. To see that this creates a particle with energy $\omega_{\mathbf{k}}$, we operate on the resulting state with $N_a(\mathbf{k})$:

$$N_a(\mathbf{k})\,a^{\dagger}(\mathbf{k})|n_{\mathbf{k}}\rangle = a^{\dagger}(\mathbf{k})\,a(\mathbf{k})\,a^{\dagger}(\mathbf{k})|n_{\mathbf{k}}\rangle \tag{5}$$

$$= a^{\dagger}(\mathbf{k})\left[1+a^{\dagger}(\mathbf{k})\,a(\mathbf{k})\right]|n_{\mathbf{k}}\rangle \tag{6}$$

$$= a^{\dagger}(\mathbf{k})\left[1+N_a(\mathbf{k})\right]|n_{\mathbf{k}}\rangle \tag{7}$$

$$= a^{\dagger}(\mathbf{k})\left[1+n_{\mathbf{k}}\right]|n_{\mathbf{k}}\rangle \tag{8}$$

$$= \left[1+n_{\mathbf{k}}\right]a^{\dagger}(\mathbf{k})|n_{\mathbf{k}}\rangle \tag{9}$$

Thus the state $a^{\dagger}(\mathbf{k})|n_{\mathbf{k}}\rangle$ is an eigenstate of $N_a(\mathbf{k})$ with eigenvalue $1+n_{\mathbf{k}}$ so the operator $a^{\dagger}(\mathbf{k})$ has created an extra $\omega_{\mathbf{k}}$ particle.

The procedure for $a(\mathbf{k})$ is similar:

$$N_a(\mathbf{k})a(\mathbf{k})|n_{\mathbf{k}}\rangle = a^{\dagger}(\mathbf{k})a(\mathbf{k})a(\mathbf{k})|n_{\mathbf{k}}\rangle \tag{10}$$

$$= \left[a(\mathbf{k})a^{\dagger}(\mathbf{k})-1\right]a(\mathbf{k})|n_{\mathbf{k}}\rangle \tag{11}$$

$$= [a(\mathbf{k})N_a(\mathbf{k})-a(\mathbf{k})]|n_{\mathbf{k}}\rangle \tag{12}$$

$$= a(\mathbf{k})[n_{\mathbf{k}}-1]|n_{\mathbf{k}}\rangle \tag{13}$$

$$= [n_{\mathbf{k}}-1]a(\mathbf{k})|n_{\mathbf{k}}\rangle \tag{14}$$

Thus the state $a(\mathbf{k})|n_{\mathbf{k}}\rangle$ is an eigenstate of $N_a(\mathbf{k})$ with eigenvalue $n_{\mathbf{k}}-1$ so the operator $a(\mathbf{k})$ has destroyed a $\omega_{\mathbf{k}}$ particle.

We can represent a state containing $n_{\mathbf{k}}$ b type particles (antiparticles) by $|\bar{n}_{\mathbf{k}}\rangle$ (a bar over a symbol means it refers to antiparticles). Then

$$N_b(\mathbf{k})|\bar{n}_{\mathbf{k}}\rangle = \bar{n}_{\mathbf{k}}|\bar{n}_{\mathbf{k}}\rangle \tag{15}$$

The two derivations above are exactly the same for antiparticles, if we replace all a operators by b operators, N_a by N_b and $n_{\mathbf{k}}$ by $\bar{n}_{\mathbf{k}}$. Thus $b^{\dagger}(\mathbf{k})$ creates a single antiparticle with energy $\omega_{\mathbf{k}}$ and $b(\mathbf{k})$ destroys one antiparticle with that energy.

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