

## CREATION AND DESTRUCTION OPERATORS FOR A FREE SCALAR FIELD

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Reference: References: Robert D. Klauber, *Student Friendly Quantum Field Theory*, (Sandtrove Press, 2013) - Chapter 3, Problem 3.10.

The scalar, free-field Hamiltonian derived earlier is

$$(1) \quad H = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \left[ N_a(\mathbf{k}) + \frac{1}{2} + N_b(\mathbf{k}) + \frac{1}{2} \right]$$

where the operators are

$$(2) \quad N_a(\mathbf{k}) = a^\dagger(\mathbf{k}) a(\mathbf{k})$$

$$(3) \quad N_b(\mathbf{k}) = b^\dagger(\mathbf{k}) b(\mathbf{k})$$

These operators can be interpreted as number operators. That is, when they operate on a state  $|n_{\mathbf{k}}\rangle$  containing  $n_{\mathbf{k}}$  particles with energy  $\omega_{\mathbf{k}}$ , their eigenvalues are just  $n_{\mathbf{k}}$ :

$$(4) \quad N_a(\mathbf{k}) |n_{\mathbf{k}}\rangle = n_{\mathbf{k}} |n_{\mathbf{k}}\rangle$$

With this interpretation, and the commutation relations for the  $a$  and  $b$  operators, we can derive the fact that  $a^\dagger(\mathbf{k})$  is a *creation operator* for type  $a$  particles with energy  $\omega_{\mathbf{k}}$ ,  $a(\mathbf{k})$  is a *destruction operator* for the same type of particle, and  $b^\dagger(\mathbf{k})$  and  $b(\mathbf{k})$  are the analogous operators for type  $b$  particles (antiparticles).

Suppose we operate on the state  $|n_{\mathbf{k}}\rangle$  with  $a^\dagger(\mathbf{k})$ . To see that this creates a particle with energy  $\omega_{\mathbf{k}}$ , we operate on the resulting state with  $N_a(\mathbf{k})$ :

$$(5) \quad N_a(\mathbf{k}) a^\dagger(\mathbf{k}) |n_{\mathbf{k}}\rangle = a^\dagger(\mathbf{k}) a(\mathbf{k}) a^\dagger(\mathbf{k}) |n_{\mathbf{k}}\rangle$$

$$(6) \quad = a^\dagger(\mathbf{k}) \left[ 1 + a^\dagger(\mathbf{k}) a(\mathbf{k}) \right] |n_{\mathbf{k}}\rangle$$

$$(7) \quad = a^\dagger(\mathbf{k}) [1 + N_a(\mathbf{k})] |n_{\mathbf{k}}\rangle$$

$$(8) \quad = a^\dagger(\mathbf{k}) [1 + n_{\mathbf{k}}] |n_{\mathbf{k}}\rangle$$

$$(9) \quad = [1 + n_{\mathbf{k}}] a^\dagger(\mathbf{k}) |n_{\mathbf{k}}\rangle$$

Thus the state  $a^\dagger(\mathbf{k})|n_{\mathbf{k}}\rangle$  is an eigenstate of  $N_a(\mathbf{k})$  with eigenvalue  $1+n_{\mathbf{k}}$  so the operator  $a^\dagger(\mathbf{k})$  has created an extra  $\omega_{\mathbf{k}}$  particle.

The procedure for  $a(\mathbf{k})$  is similar:

$$(10) \quad N_a(\mathbf{k})a(\mathbf{k})|n_{\mathbf{k}}\rangle = a^\dagger(\mathbf{k})a(\mathbf{k})a(\mathbf{k})|n_{\mathbf{k}}\rangle$$

$$(11) \quad = [a(\mathbf{k})a^\dagger(\mathbf{k}) - 1]a(\mathbf{k})|n_{\mathbf{k}}\rangle$$

$$(12) \quad = [a(\mathbf{k})N_a(\mathbf{k}) - a(\mathbf{k})]|n_{\mathbf{k}}\rangle$$

$$(13) \quad = a(\mathbf{k})[n_{\mathbf{k}} - 1]|n_{\mathbf{k}}\rangle$$

$$(14) \quad = [n_{\mathbf{k}} - 1]a(\mathbf{k})|n_{\mathbf{k}}\rangle$$

Thus the state  $a(\mathbf{k})|n_{\mathbf{k}}\rangle$  is an eigenstate of  $N_a(\mathbf{k})$  with eigenvalue  $n_{\mathbf{k}} - 1$  so the operator  $a(\mathbf{k})$  has destroyed a  $\omega_{\mathbf{k}}$  particle.

We can represent a state containing  $n_{\mathbf{k}}$   $b$  type particles (antiparticles) by  $|\bar{n}_{\mathbf{k}}\rangle$  (a bar over a symbol means it refers to antiparticles). Then

$$(15) \quad N_b(\mathbf{k})|\bar{n}_{\mathbf{k}}\rangle = \bar{n}_{\mathbf{k}}|\bar{n}_{\mathbf{k}}\rangle$$

The two derivations above are exactly the same for antiparticles, if we replace all  $a$  operators by  $b$  operators,  $N_a$  by  $N_b$  and  $n_{\mathbf{k}}$  by  $\bar{n}_{\mathbf{k}}$ . Thus  $b^\dagger(\mathbf{k})$  creates a single antiparticle with energy  $\omega_{\mathbf{k}}$  and  $b(\mathbf{k})$  destroys one antiparticle with that energy.

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