

REAL KLEIN-GORDON FIELDS

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Reference: References: Robert D. Klauber, *Student Friendly Quantum Field Theory*, (Sandtrove Press, 2013) - Chapter 3, Problem 3.15.

When we derived the Klein-Gordon equation in field theory, we assumed that the field ϕ is complex, so that both the field itself and its complex conjugate ϕ^\dagger are viewed as independent fields. However, we can also take the field ϕ to be real, in which case there is only a single field. The Lagrangian in this case is the same as the one we used originally:

$$\mathcal{L}_0^0 = K (\partial_\alpha \phi \partial^\alpha \phi - \mu^2 \phi^2) \quad (1)$$

The superscript 0 on \mathcal{L}_0^0 indicates that we're dealing with a *scalar* field (as opposed to a field with spin) and the subscript 0 indicates that it's a *free* field (no potential terms). K is a constant. Written out in terms of time and space derivatives, this is

$$\mathcal{L}_0^0 = K (\dot{\phi}^2 - (\nabla\phi) \cdot (\nabla\phi) - \mu^2 \phi^2) \quad (2)$$

In the complex field case, we defined the conjugate momentum density as

$$\pi_r \equiv \frac{\partial \mathcal{L}}{\partial \dot{\phi}^r} \quad (3)$$

The complex field Lagrangian takes $K = 1$ and combines the field and its complex conjugate to give a real Lagrangian:

$$\mathcal{L}_0^0 = \dot{\phi}^\dagger \dot{\phi} - \nabla\phi^\dagger \cdot \nabla\phi - \mu^2 \phi^\dagger \phi \quad (4)$$

This gives a conjugate momentum density of

$$\pi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \dot{\phi}^\dagger \quad (5)$$

In the real field case, taking $K = 1$ and using the same definition of conjugate momentum density gives $\pi = 2\dot{\phi}$. If we require the conjugate momentum density to have the same form 5 in the real field case (where $\dot{\phi}^\dagger = \dot{\phi}$ since ϕ is real), then we must take $K = \frac{1}{2}$, since from 2 we have

$$\pi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = 2K\dot{\phi} = \dot{\phi} \quad (6)$$