FEYNMAN PROPAGATOR FOR SCALAR FIELDS

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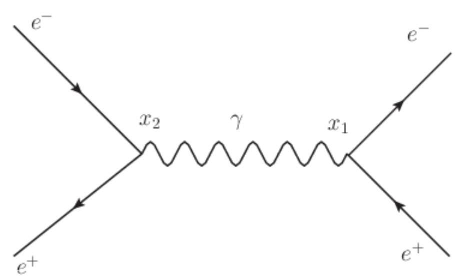
Reference: References: Robert D. Klauber, *Student Friendly Quantum Field Theory*, (Sandtrove Press, 2013) - Chapter 3, Problem 3.17.

In quantum field theory, interactions between particles are mediated by *virtual particles*, which are particles that are never observed, but which carry the information from the particles before the interaction to the particles after the interaction. A particle interaction can be represented graphically by a Feynman diagram. One example is this:



In the diagram, time increases from left to right. The arrow on a line indicates the motion of a particle, and rather confusingly at first, antiparticles are shown with arrows opposite to their direction of propagation. Thus in this interaction, an electron e^- and a positron (antielectron) e^+ move in from the left and meet at point x_2. They annihilate each other, producing a photon (the wavy line, labelled γ for gamma particle). The photon is a virtual particle (it is never observed in an experiment) which propagates to location x_1 where it spontaneously dissociates into an electron-positron pair which move off to the right.

The physics of the virtual particle, the photon in this case, is described by a *Feynman propagator*, or just *propagator*. In its simplest form, a propagator creates a virtual particle from the vacuum and, a short time later, annihilates it. We can use the Klein-Gordon fields derived earlier to see how this works. [Note that a photon is not described by a Klein-Gordon field, since the photon has spin 1 and is not a scalar particle.]

The continuous fields are

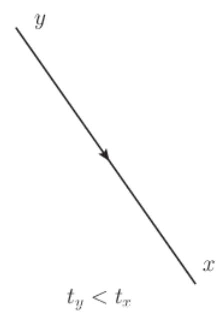
$$\phi(x) = \int \frac{d^3k}{\sqrt{2(2\pi)^3\omega_{\mathbf{k}}}} a(\mathbf{k})\, e^{-ikx} + \int \frac{d^3k}{\sqrt{2(2\pi)^3\omega_{\mathbf{k}}}} b^\dagger(\mathbf{k})\, e^{ikx} \tag{1}$$

$$\equiv \phi^+ + \phi^- \tag{2}$$

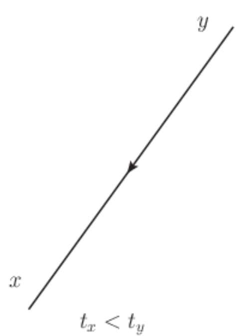
$$\phi^\dagger(x) = \int \frac{d^3k}{\sqrt{2(2\pi)^3\omega_{\mathbf{k}}}} b(\mathbf{k})\, e^{-ikx} + \int \frac{d^3k}{\sqrt{2(2\pi)^3\omega_{\mathbf{k}}}} a^\dagger(\mathbf{k})\, e^{ikx} \tag{3}$$

$$\equiv \phi^{\dagger +} + \phi^{\dagger -} \tag{4}$$

For a scalar field, there are two situations. First, we can create a particle at location $\mathbf{y}$ at time t_y, then at a later time $t_x > t_y$, we can annihilate the same particle at location $\mathbf{x}$. This is shown in the Feynman diagram:



Second, if $t_x < t_y$ we can create an antiparticle at $\mathbf{x}$ and annihilate it at $\mathbf{y}$, as shown



The important point is that these two virtual particle situations have the same result in an experiment. If a virtual particle is created at $\mathbf{y}$ at t_y and travels to $\mathbf{x}$ at time t_x, it carries information (charge and so on) from $\mathbf{y}$ to $\mathbf{x}$. If the corresponding virtual *anti*particle is created at $\mathbf{x}$ at t_x and travels to $\mathbf{y}$ at t_y, it carries exactly the opposite information (since it's an antiparticle) from $\mathbf{x}$ to $\mathbf{y}$. Thus in a real experiment, the propagator must include both possibilities.

Klauber treats the case of $t_y < t_x$, so we'll look at the other case (the two derivations are very similar). That is, we want to create an antiparticle at x and annihilate it at y. From the equations above, we see that ϕ creates antiparticles (it contains the $b^\dagger$ operators) and $\phi^\dagger$ destroys them (it contains the b operators), so the life of the virtual particle is described by applying these two fields in some order. Since we want to create an antiparticle first and *then* annihilate it, we need to apply ϕ first, then $\phi^\dagger$. The situation is reversed if we want to create a particle and then annihilate it, since in that case $\phi^\dagger$ contains the $a^\dagger$ operators and ϕ contains the a operators. The two time orders above thus require the fields to be applied in different orders, and a *time ordering operator* T is defined so that

$$T\left[\phi(x)\phi^\dagger(y)\right]=\begin{cases}\phi(x)\phi^\dagger(y) & \text{if } t_y<t_x\\ \phi^\dagger(y)\phi(x) & \text{if } t_x<t_y\end{cases} \tag{5}$$

We're interested in the second case. The *transition amplitude* for a process in which a virtual particle is created out of the vacuum and then decays back into the vacuum is then

$$\left\langle 0\middle|T\left[\phi(x)\phi^\dagger(y)\right]\middle|0\right\rangle \tag{6}$$

Looking at the antiparticle case, we have

$$T\left[\phi(x)\phi^\dagger(y)\right]|0\rangle=\left(\phi^{\dagger+}(y)+\phi^{\dagger-}(y)\right)\left(\phi^+(x)+\phi^-(x)\right)|0\rangle \tag{7}$$

If we're looking at an antiparticle with a specific wave number $\mathbf{k}$, then ϕ^- creates an antiparticle and $\phi^{\dagger+}$ destroys an antiparticle (while ϕ^+ destroys a particle and $\phi^{\dagger-}$ creates a particle). Any annihilation operator acting on the vacuum gives zero, so

$$\left(\phi^+(x)+\phi^-(x)\right)|0\rangle = \left(0+\phi^-(x)\right)|0\rangle \tag{8}$$

$$= A(x)\left|\bar{\phi}\right\rangle \tag{9}$$

where $A(x)$ is a numerical function (not an operator), since a creation operator acting on the vacuum gives a number (determined by normalization) multiplied by the state $\left|\bar{\phi}\right\rangle$ containing a single antiparticle.

Returning to 7, we see that operating on this result with $\phi^{\dagger-}(y)$ creates a particle, so gives the state $\left|\bar{\phi}\phi\right\rangle$ multiplied by some other numerical function $B(y)$, while operating on $A(x)\left|\bar{\phi}\right\rangle$ with $\phi^{\dagger+}(y)$ destroys the antiparticle just created, producing the vacuum state $|0\rangle$ multiplied by some other numerical function $C(y)$. Therefore we get

$$T\left[\phi(x)\phi^{\dagger}(y)\right]|0\rangle = C(y)A(x)|0\rangle + B(y)A(x)\left|\bar{\phi}\phi\right\rangle \tag{10}$$

Thus the transition amplitude 6 is

$$\left\langle 0\left|T\left[\phi(x)\phi^{\dagger}(y)\right]\right|0\right\rangle = \langle 0|C(y)A(x)|0\rangle + \left\langle 0|B(y)A(x)|\bar{\phi}\phi\right\rangle \tag{11}$$

The brackets imply an integration over all space, but we're interested in the antiparticle creation occurring at a specific location x and annihilation at another specific location y, so these two locations are actually constants relative to the integration variable, and can come outside the brackets. From the orthonormality of quantum states, $\langle 0|0\rangle = 1$ and $\left\langle 0\left|\bar{\phi}\phi\right.\right\rangle = 0$, so we get

$$\left\langle 0\left|T\left[\phi(x)\phi^{\dagger}(y)\right]\right|0\right\rangle = C(y)A(x) \tag{12}$$

The result for creating and annihilating a particle (as opposed to an antiparticle) is the same, although the numerical functions can be different. Klauber calls them $G(x)$ and $F(y)$, so that for the particle case

$$\left\langle 0\left|T\left[\phi(x)\phi^{\dagger}(y)\right]\right|0\right\rangle = F(y)G(x) \tag{13}$$

The vacuum expectation value of the time ordering operator is called the *Feynman propagator*, defined as $i\Delta_F(x-y)$:

$$i\Delta_F(x-y) \equiv \left\langle 0\left|T\left[\phi(x)\phi^{\dagger}(y)\right]\right|0\right\rangle \tag{14}$$

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