

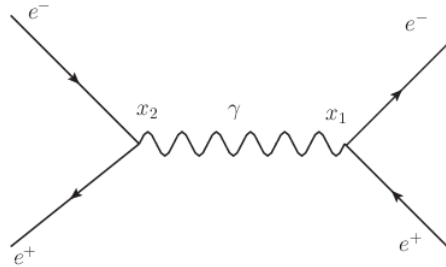
FEYNMAN PROPAGATOR FOR SCALAR FIELDS

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Reference: References: Robert D. Klauber, *Student Friendly Quantum Field Theory*, (Sandtrove Press, 2013) - Chapter 3, Problem 3.17.

In quantum field theory, interactions between particles are mediated by *virtual particles*, which are particles that are never observed, but which carry the information from the particles before the interaction to the particles after the interaction. A particle interaction can be represented graphically by a Feynman diagram. One example is this:



In the diagram, time increases from left to right. The arrow on a line indicates the motion of a particle, and rather confusingly at first, antiparticles are shown with arrows opposite to their direction of propagation. Thus in this interaction, an electron e^- and a positron (antielectron) e^+ move in from the left and meet at point x_2 . They annihilate each other, producing a photon (the wavy line, labelled γ for gamma particle). The photon is a virtual particle (it is never observed in an experiment) which propagates to location x_1 where it spontaneously dissociates into an electron-positron pair which move off to the right.

The physics of the virtual particle, the photon in this case, is described by a *Feynman propagator*, or just *propagator*. In its simplest form, a propagator creates a virtual particle from the vacuum and, a short time later, annihilates it. We can use the Klein-Gordon fields derived earlier to see how this works. [Note that a photon is not described by a Klein-Gordon field, since the photon has spin 1 and is not a scalar particle.]

The continuous fields are

$$(1) \quad \phi(x) = \int \frac{d^3k}{\sqrt{2(2\pi)^3 \omega_{\mathbf{k}}}} a(\mathbf{k}) e^{-ikx} + \int \frac{d^3k}{\sqrt{2(2\pi)^3 \omega_{\mathbf{k}}}} b^\dagger(\mathbf{k}) e^{ikx}$$

$$(2) \quad \equiv \phi^+ + \phi^-$$

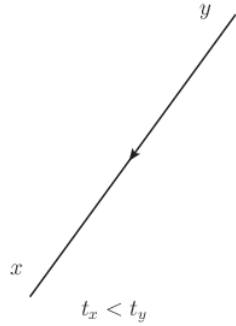
$$(3) \quad \phi^\dagger(x) = \int \frac{d^3k}{\sqrt{2(2\pi)^3 \omega_{\mathbf{k}}}} b(\mathbf{k}) e^{-ikx} + \int \frac{d^3k}{\sqrt{2(2\pi)^3 \omega_{\mathbf{k}}}} a^\dagger(\mathbf{k}) e^{ikx}$$

$$(4) \quad \equiv \phi^{\dagger+} + \phi^{\dagger-}$$

For a scalar field, there are two situations. First, we can create a particle at location \mathbf{y} at time t_y , then at a later time $t_x > t_y$, we can annihilate the same particle at location \mathbf{x} . This is shown in the Feynman diagram:



Second, if $t_x < t_y$ we can create an antiparticle at \mathbf{x} and annihilate it at \mathbf{y} , as shown



The important point is that these two virtual particle situations have the same result in an experiment. If a virtual particle is created at \mathbf{y} at t_y and travels to \mathbf{x} at time t_x , it carries information (charge and so on) from \mathbf{y} to \mathbf{x} . If the corresponding virtual *antiparticle* is created at \mathbf{x} at t_x and travels to \mathbf{y} at t_y , it carries exactly the opposite information (since it's an antiparticle) from \mathbf{x} to \mathbf{y} . Thus in a real experiment, the propagator must include both possibilities.

Klauber treats the case of $t_y < t_x$, so we'll look at the other case (the two derivations are very similar). That is, we want to create an antiparticle at x and annihilate it at y . From the equations above, we see that ϕ creates antiparticles (it contains the b^\dagger operators) and ϕ^\dagger destroys them (it contains the b operators), so the life of the virtual particle is described by applying these two fields in some order. Since we want to create an antiparticle first and *then* annihilate it, we need to apply ϕ first, then ϕ^\dagger . The situation is reversed if we want to create a particle and then annihilate it, since in that case ϕ^\dagger contains the a^\dagger operators and ϕ contains the a operators. The two time orders above thus require the fields to be applied in different orders, and a *time ordering operator* T is defined so that

$$(5) \quad T \left[\phi(x) \phi^\dagger(y) \right] = \begin{cases} \phi(x) \phi^\dagger(y) & \text{if } t_y < t_x \\ \phi^\dagger(y) \phi(x) & \text{if } t_x < t_y \end{cases}$$

We're interested in the second case. The *transition amplitude* for a process in which a virtual particle is created out of the vacuum and then decays back into the vacuum is then

$$(6) \quad \langle 0 | T \left[\phi(x) \phi^\dagger(y) \right] | 0 \rangle$$

Looking at the antiparticle case, we have

$$(7) \quad T \left[\phi(x) \phi^\dagger(y) \right] | 0 \rangle = \left(\phi^{\dagger+}(y) + \phi^{\dagger-}(y) \right) \left(\phi^+(x) + \phi^-(x) \right) | 0 \rangle$$

If we're looking at an antiparticle with a specific wave number \mathbf{k} , then ϕ^- creates an antiparticle and $\phi^{\dagger+}$ destroys an antiparticle (while ϕ^+ destroys a particle and $\phi^{\dagger-}$ creates a particle). Any annihilation operator acting on the vacuum gives zero, so

$$(8) \quad \left(\phi^+(x) + \phi^-(x) \right) | 0 \rangle = \left(0 + \phi^-(x) \right) | 0 \rangle$$

$$(9) \quad = A(x) | \bar{\phi} \rangle$$

where $A(x)$ is a numerical function (not an operator), since a creation operator acting on the vacuum gives a number (determined by normalization) multiplied by the state $|\bar{\phi}\rangle$ containing a single antiparticle.

Returning to 7, we see that operating on this result with $\phi^{\dagger-}(y)$ creates a particle, so gives the state $|\bar{\phi}\phi\rangle$ multiplied by some other numerical function $B(y)$, while operating on $A(x)|\bar{\phi}\rangle$ with $\phi^{\dagger+}(y)$ destroys the antiparticle just created, producing the vacuum state $|0\rangle$ multiplied by some other numerical function $C(y)$. Therefore we get

$$(10) \quad T \left[\phi(x) \phi^\dagger(y) \right] |0\rangle = C(y) A(x) |0\rangle + B(y) A(x) |\bar{\phi}\phi\rangle$$

Thus the transition amplitude 6 is

$$(11) \quad \langle 0 | T \left[\phi(x) \phi^\dagger(y) \right] | 0 \rangle = \langle 0 | C(y) A(x) | 0 \rangle + \langle 0 | B(y) A(x) | \bar{\phi}\phi \rangle$$

The brackets imply an integration over all space, but we're interested in the antiparticle creation occurring at a specific location x and annihilation at another specific location y , so these two locations are actually constants relative to the integration variable, and can come outside the brackets. From the orthonormality of quantum states, $\langle 0 | 0 \rangle = 1$ and $\langle 0 | \bar{\phi}\phi \rangle = 0$, so we get

$$(12) \quad \langle 0 | T \left[\phi(x) \phi^\dagger(y) \right] | 0 \rangle = C(y) A(x)$$

The result for creating and annihilating a particle (as opposed to an antiparticle) is the same, although the numerical functions can be different. Klauber calls them $G(x)$ and $F(y)$, so that for the particle case

$$(13) \quad \langle 0 | T \left[\phi(x) \phi^\dagger(y) \right] | 0 \rangle = F(y) G(x)$$

The vacuum expectation value of the time ordering operator is called the *Feynman propagator*, defined as $i\Delta_F(x-y)$:

$$(14) \quad i\Delta_F(x-y) \equiv \langle 0 | T \left[\phi(x) \phi^\dagger(y) \right] | 0 \rangle$$

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