ADJOINT DIRAC EQUATION

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

Reference: References: Robert D. Klauber, *Student Friendly Quantum Field Theory*, (Sandtrove Press, 2013) - Chapter 4, Problem 4.11.

The Dirac equation in condensed form is

$$i\gamma^{\mu}\partial_{\mu}|\psi\rangle = m|\psi\rangle \tag{1}$$

where the gamma matrices have been defined earlier. The Hermitian conjugate of the gamma matrix γ^{μ} is given by the Hermiticity condition

$$\gamma^{\mu\dagger} = \gamma^{0}\gamma^{\mu}\gamma^{0} \tag{2}$$

To get the adjoint form of the Dirac equation, we use the adjoint solutions

$$\left\langle \bar{\psi}^{(n)}\right| \equiv \left\langle \psi^{(n)}\right|\gamma^{0} \tag{3}$$

The Hermitian conjugate of 1 is

$$-i\partial_{\mu}\langle\psi|\gamma^{\mu\dagger} = m\langle\psi| \tag{4}$$

Remember that when we take the Hermitian conjugate of a matrix equation we must take the Hermitian conjugate of each matrix and also reverse the order of matrix multiplication, which is why the $\gamma^{\mu\dagger}$ term appears at the end on the LHS. The ∂_{μ} is a differential operator, not a matrix, so it retains its position in front of the $\langle\psi|$.

Using 2, we can write this as

$$-i\partial_{\mu}\langle\psi|\gamma^{0}\gamma^{\mu}\gamma^{0} = m\langle\psi| \tag{5}$$

Then, by post-multiplying by γ^{0} and using $\left(\gamma^{0}\right)^{2} = I$, the identity matrix, we get, using the definition 3

$$\begin{aligned}
-i\partial_{\mu}\langle\psi|\gamma^{0}\gamma^{\mu}\left(\gamma^{0}\right)^{2} &= m\langle\psi|\gamma^{0} && (6)\\
-i\partial_{\mu}\langle\bar{\psi}|\gamma^{\mu} &= m\langle\bar{\psi}| && (7)\\
i\partial_{\mu}\langle\bar{\psi}|\gamma^{\mu} + m\langle\bar{\psi}| &= 0 && (8)
\end{aligned}$$

Note that although the terms $\langle\bar{\psi}|$ in this adjoint equation are adjoint solutions, the gamma matrices γ^{μ} are the *original* (that is, *not* the Hermitian conjugate) gamma matrices.

PINGBACKS

Pingback: Adjoint Dirac equation: explicit solutions
Pingback: Dirac equation: conserved probability current
Pingback: Dirac equation in relativistic quantum mechanics: summary
Pingback: Dirac equation in quantum field theory: Lagrangian density