

ADJOINT DIRAC EQUATION

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

Reference: References: Robert D. Klauber, *Student Friendly Quantum Field Theory*, (Sandtrove Press, 2013) - Chapter 4, Problem 4.11.

The Dirac equation in condensed form is

$$i\gamma^\mu \partial_\mu |\psi\rangle = m|\psi\rangle \quad (1)$$

where the gamma matrices have been defined earlier. The Hermitian conjugate of the gamma matrix γ^μ is given by the Hermiticity condition

$$\gamma^{\mu\dagger} = \gamma^0 \gamma^\mu \gamma^0 \quad (2)$$

To get the adjoint form of the Dirac equation, we use the adjoint solutions

$$\langle \bar{\psi}^{(n)} | \equiv \langle \psi^{(n)} | \gamma^0 \quad (3)$$

The Hermitian conjugate of 1 is

$$-i\partial_\mu \langle \psi | \gamma^{\mu\dagger} = m \langle \psi | \quad (4)$$

Remember that when we take the Hermitian conjugate of a matrix equation we must take the Hermitian conjugate of each matrix and also reverse the order of matrix multiplication, which is why the $\gamma^{\mu\dagger}$ term appears at the end on the LHS. The ∂_μ is a differential operator, not a matrix, so it retains its position in front of the $\langle \psi |$.

Using 2, we can write this as

$$-i\partial_\mu \langle \psi | \gamma^0 \gamma^\mu \gamma^0 = m \langle \psi | \quad (5)$$

Then, by post-multiplying by γ^0 and using $(\gamma^0)^2 = I$, the identity matrix, we get, using the definition 3

$$-i\partial_\mu \langle \psi | \gamma^0 \gamma^\mu (\gamma^0)^2 = m \langle \psi | \gamma^0 \quad (6)$$

$$-i\partial_\mu \langle \bar{\psi} | \gamma^\mu = m \langle \bar{\psi} | \quad (7)$$

$$i\partial_\mu \langle \bar{\psi} | \gamma^\mu + m \langle \bar{\psi} | = 0 \quad (8)$$

Note that although the terms $\langle \bar{\psi} |$ in this adjoint equation are adjoint solutions, the gamma matrices γ^μ are the *original* (that is, *not* the Hermitian conjugate) gamma matrices.

PINGBACKS

[Pingback: Adjoint Dirac equation: explicit solutions](#)

[Pingback: Dirac equation: conserved probability current](#)

[Pingback: Dirac equation in relativistic quantum mechanics: summary](#)

[Pingback: Dirac equation in quantum field theory: Lagrangian density](#)