

## NATURAL UNITS: THE MUON LIFETIME

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References: Amitabha Lahiri & P. B. Pal, *A First Book of Quantum Field Theory*, Second Edition (Alpha Science International, 2004) - Chapter 1, Problem 1.8.

In natural units  $\hbar = 1$  and  $c = 1$  which results in every physical quantity being expressed in units of mass. As an example, the muon lifetime  $\tau$  is given by

$$\tau^{-1} = \frac{G_F^2 m^5}{192\pi^3} \quad (1)$$

where  $m$  is the muon mass which is 106 MeV and  $G_F$  is the Fermi coupling constant, which comes out of the quantum field theory of the electroweak interaction (more on this [much] later, hopefully). Since time has natural units of inverse mass  $M^{-1}$ , the units of  $G_F$  must be  $M^{-2}$  to make the units balance out.

To convert this formula into SI units, we need to insert factors of  $\hbar$  and  $c$  so that the LHS has units of  $s^{-1}$ . Since the units are currently  $M^1$  where mass is expressed in MeV, which is an energy unit, and the units of  $\hbar$  are those of action, which is (energy)  $\times$  (time), we can divide by  $\hbar$  to get overall units of (time) $^{-1}$ . Thus

$$\tau^{-1} = \frac{G_F^2 m^5}{192\pi^3 \hbar} \quad (2)$$

Given that  $G_F = 1.166 \times 10^{-11} \text{ MeV}^{-2}$  and  $\hbar = 6.58 \times 10^{-22} \text{ MeV s}$ , we get

$$\tau^{-1} = 4.645 \times 10^5 \text{ s}^{-1} \quad (3)$$

$$\tau = 2.15 \times 10^{-6} \text{ s} \quad (4)$$