HAMILTONIAN FOR COMPLEX SCALAR FIELD

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References: Amitabha Lahiri & P. B. Pal, *A First Book of Quantum Field Theory*, Second Edition (Alpha Science International, 2004) - Chapter 2, Problem 2.6.

Here we revisit the complex scalar field we considered earlier. The Lagrangian density is

$$\mathscr{L}=\left(\partial_{\mu}\phi\right)^{\dagger}\left(\partial^{\mu}\phi\right)-m^{2}\phi^{\dagger}\phi-V\left(\phi^{\dagger}\phi\right) \tag{1}$$

where ϕ is the complex field, and the potential V is a function of $\phi^{\dagger}\phi$, so it is a real function. The Hamiltonian density is then defined as

$$\mathscr{H}\equiv\Pi_{A}\dot{\Phi}^{A}-\mathscr{L} \tag{2}$$

where Π_{A} is the conjugate momentum for field Φ^{A}, defined as

$$\Pi_{A}\equiv\frac{\delta L}{\delta\dot{\Phi}^{A}} \tag{3}$$

In our case, there are two fields, ϕ and $\phi^{\dagger}$. We can rewrite 1 as

$$\mathscr{L}=\dot{\phi}^{\dagger}\dot{\phi}-\left|\nabla\phi\right|^{2}-m^{2}\phi^{\dagger}\phi-V\left(\phi^{\dagger}\phi\right) \tag{4}$$

so the conjugate momenta are

$$\Pi=\int d^{3}y\dot{\phi}^{\dagger}\left(\mathbf{y},t\right)\delta\left(\mathbf{x}-\mathbf{y}\right) \tag{5}$$

$$=\dot{\phi}^{\dagger}\left(\mathbf{x},t\right) \tag{6}$$

$$\Pi^{\dagger}=\dot{\phi}\left(\mathbf{x},t\right) \tag{7}$$

The Hamiltonian density is therefore

$$\mathscr{H}=\dot{\phi}^{\dagger}\dot{\phi}+\dot{\phi}\dot{\phi}^{\dagger}-\left(\dot{\phi}^{\dagger}\dot{\phi}-\left|\nabla\phi\right|^{2}-m^{2}\phi^{\dagger}\phi-V\left(\phi^{\dagger}\phi\right)\right) \tag{8}$$

$$=\dot{\phi}^{\dagger}\dot{\phi}+\left|\nabla\phi\right|^{2}+m^{2}\phi^{\dagger}\phi+V\left(\phi^{\dagger}\phi\right) \tag{9}$$

$$=\left|\dot{\phi}\right|^{2}+\left|\nabla\phi\right|^{2}+m^{2}\phi^{\dagger}\phi+V\left(\phi^{\dagger}\phi\right) \tag{10}$$