FEYNMAN PROPAGATOR FOR A COMPLEX SCALAR FIELD: TIME ORDERING

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References: Amitabha Lahiri & P. B. Pal, *A First Book of Quantum Field Theory*, Second Edition (Alpha Science International, 2004) - Chapter 3, Problem 3.10.

The Feynman propagator for a scalar field turns out to be

$$i\Delta_F\left(x-x'\right)=\int\frac{d^3p}{(2\pi)^3\,2E_p}\left[\Theta\left(t-t'\right)e^{-ip\cdot(x-x')}+\Theta\left(t'-t\right)e^{ip\cdot(x-x')}\right] \tag{1}$$

By applying the formula for the field in terms of creation and annihilation operators to the vacuum state of a real scalar field, L&P show that we can convert the Feynman propagator into a form involving the fields:

$$i\Delta_F\left(x-x'\right)=\Theta\left(t-t'\right)\left\langle 0\left|\phi\left(x\right)\phi\left(x'\right)\right|0\right\rangle+\Theta\left(t'-t\right)\left\langle 0\left|\phi\left(x'\right)\phi\left(x\right)\right|0\right\rangle \tag{2}$$

where

$$\Theta\left(t-t'\right)=\begin{cases}1 & \text{if } t>t'\\ \frac{1}{2} & \text{if } t=t'\\ 0 & \text{if } t<t'\end{cases} \tag{3}$$

This can be written in a more compact form as

$$i\Delta_F\left(x-x'\right)=\left\langle 0\left|\mathscr{T}\left[\phi\left(x\right)\phi\left(x'\right)\right]\right|0\right\rangle \tag{4}$$

where $\mathscr{T}$ is the time-ordering operator, which orders its arguments so that the time increases from right to left.

We can follow through a similar argument to that in L&P leading to equation 3.82 for a complex scalar field. In this case, the fields are

$$\phi(x)=\int\frac{d^3p}{\sqrt{(2\pi)^3 2E_p}}\left(a(p)e^{-ip\cdot x}+\widehat{a}^\dagger(p)e^{ip\cdot x}\right) \tag{5}$$

$$\phi^\dagger(x)=\int\frac{d^3p}{\sqrt{(2\pi)^3 2E_p}}\left(\widehat{a}(p)e^{-ip\cdot x}+a^\dagger(p)e^{ip\cdot x}\right) \tag{6}$$

If we apply these fields to the vacuum state $|0\rangle$, only those terms with creation operators will have any effect, so we have

$$\phi(x)|0\rangle=\int\frac{d^3p}{\sqrt{(2\pi)^3 2E_p}}e^{ip\cdot x}\widehat{a}^\dagger(p)|0\rangle \tag{7}$$

$$=\int\frac{d^3p}{\sqrt{(2\pi)^3 2E_p}}e^{ip\cdot x}|\widehat{p}\rangle \tag{8}$$

where $|\widehat{p}\rangle$ is the state with a single antiparticle with momentum p. Similarly, we have

$$\phi^\dagger(x)|0\rangle=\int\frac{d^3p}{\sqrt{(2\pi)^3 2E_p}}e^{ip\cdot x}a^\dagger(p)|0\rangle \tag{9}$$

$$=\int\frac{d^3p}{\sqrt{(2\pi)^3 2E_p}}e^{ip\cdot x}|p\rangle \tag{10}$$

For the bra terms, only the annihilation operators (acting to the left) contribute, so we have

$$\langle 0|\phi(x)=\int\frac{d^3p}{\sqrt{(2\pi)^3 2E_p}}\langle 0|a(p)e^{-ip\cdot x} \tag{11}$$

$$=\int\frac{d^3p}{\sqrt{(2\pi)^3 2E_p}}\langle p|e^{-ip\cdot x} \tag{12}$$

$$\langle 0|\phi^\dagger(x)=\int\frac{d^3p}{\sqrt{(2\pi)^3 2E_p}}\langle 0|\widehat{a}(p)e^{-ip\cdot x} \tag{13}$$

$$=\int\frac{d^3p}{\sqrt{(2\pi)^3 2E_p}}\langle\widehat{p}|e^{-ip\cdot x} \tag{14}$$

Combining these results using the normalizations

$$\left\langle p\left|p'\right.\right\rangle = \left\langle \widehat{p}\left|\widehat{p}'\right.\right\rangle = \delta^3\left(\mathbf{p}-\mathbf{p}'\right) \tag{15}$$

and doing the integral over d^3p' we have

$$\left\langle 0\left|\phi^\dagger\left(x'\right)\phi\left(x\right)\right|0\right\rangle = \int\frac{d^3p}{(2\pi)^3\,2E_p}e^{ip\cdot(x-x')} \tag{16}$$

$$\left\langle 0\left|\phi\left(x\right)\phi^\dagger\left(x'\right)\right|0\right\rangle = \int\frac{d^3p}{(2\pi)^3\,2E_p}e^{-ip\cdot(x-x')} \tag{17}$$

Comparing with 1, we can write the propagator as

$$i\Delta_F\left(x-x'\right) = \Theta\left(t-t'\right)\left\langle 0\left|\phi\left(x\right)\phi^\dagger\left(x'\right)\right|0\right\rangle + \Theta\left(t'-t\right)\left\langle 0\left|\phi^\dagger\left(x'\right)\phi\left(x\right)\right|0\right\rangle \tag{18}$$

$$= \left\langle 0\left|\mathscr{T}\left[\phi\left(x\right)\phi^\dagger\left(x'\right)\right]\right|0\right\rangle \tag{19}$$

We can also write the terms in another way, so we have

$$\left\langle 0\left|\phi\left(x'\right)\phi^\dagger\left(x\right)\right|0\right\rangle = \int\frac{d^3p}{(2\pi)^3\,2E_p}e^{ip\cdot(x-x')} \tag{20}$$

$$\left\langle 0\left|\phi^\dagger\left(x\right)\phi\left(x'\right)\right|0\right\rangle = \int\frac{d^3p}{(2\pi)^3\,2E_p}e^{-ip\cdot(x-x')} \tag{21}$$

so we also have

$$i\Delta_F\left(x-x'\right) = \Theta\left(t-t'\right)\left\langle 0\left|\phi^\dagger\left(x\right)\phi\left(x'\right)\right|0\right\rangle + \Theta\left(t'-t\right)\left\langle 0\left|\phi\left(x'\right)\phi^\dagger\left(x\right)\right|0\right\rangle \tag{22}$$

$$= \left\langle 0\left|\mathscr{T}\left[\phi\left(x'\right)\phi^\dagger\left(x\right)\right]\right|0\right\rangle \tag{23}$$

The interpretation of these results is that, in 19, the field operator $\phi^\dagger$ creates a particle (as opposed to an antiparticle; see 10) at time t' (if $t'<t$) and the ϕ operator on the left annihilates the particle at the later time t. If $t'>t$, then the ϕ field acts first and creates an antiparticle (see 5) which gets annihilated at the later time t'.

The result 23 just propagates a particle or antiparticle, but with the times reversed from 19.