

DIRAC EQUATION: SET OF INDEPENDENT MATRICES

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References: Amitabha Lahiri & P. B. Pal, *A First Book of Quantum Field Theory*, Second Edition (Alpha Science International, 2004) - Chapter 4, Problem 4.2.

In the Dirac equation, we introduced 5 γ matrices labelled γ^μ and γ^5 . These matrices satisfied the properties

$$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 \quad (1)$$

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \quad (2)$$

$$(\gamma^0)^2 = 1 \quad (3)$$

$$(\gamma^i)^2 = -1 \quad (4)$$

$$\text{Tr } \gamma^\mu = 0 \quad (5)$$

$$\text{Tr } \gamma^5 = 0 \quad (6)$$

$$\{\gamma^\mu, \gamma^5\} = 0 \quad (7)$$

$$(\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0 \quad (8)$$

We also introduced the matrices

$$\sigma^{\mu\nu} \equiv \frac{i}{2}[\gamma^\mu, \gamma^\nu] = -\sigma^{\nu\mu} \quad (9)$$

We can form the set of 16 matrices defined as

$$\Gamma = \{1, \gamma^\mu, \sigma^{\mu\nu}, \gamma^5\gamma^\mu, \gamma^5\} \quad (10)$$

We want to show that the product of any two Γ_r s is a linear combination of the existing Γ_r s.

The first thing to notice is that the elements of Γ as listed comprise matrices consisting of products of zero (that is, the unit matrix), one, two, three and four of the γ^μ . This is obvious for 1, γ^μ , $\sigma^{\mu\nu}$ and γ^5 . To see that $\gamma^5\gamma^\mu$ is equivalent to the product of 3 γ^ν s, use the anticommutation rules to swap γ^μ into γ^5 until it is adjacent to the γ^ν with $\nu = \mu$. We can now use 2 with

$\mu = \nu$ to see that $(\gamma^\mu)^2 = \pm 1$, so the product $\gamma^5 \gamma^\mu$ reduces to the product of the three remaining γ^ν s (times i).

In fact, we can use the same procedure to condense the product of any number of γ^μ s down to a product of 4 or fewer γ^ν s since, if there are more than 4 matrices in the product, some of them must be the same so by a process of swapping matrices using the anticommutation relations we can always bring equal matrices next to each other where they can be removed using $(\gamma^\mu)^2 = \pm 1$.

In passing, we can also note that, from 1e

$$(\gamma^5)^2 = -\gamma^0 \gamma^1 \gamma^2 \gamma^3 \gamma^0 \gamma^1 \gamma^2 \gamma^3 \quad (11)$$

$$= -(-1)^3 (\gamma^0)^2 \gamma^1 \gamma^2 \gamma^3 \gamma^1 \gamma^2 \gamma^3 \quad (12)$$

$$= \gamma^1 \gamma^2 \gamma^3 \gamma^1 \gamma^2 \gamma^3 \quad (13)$$

$$= (-1)^2 (\gamma^1)^2 \gamma^2 \gamma^3 \gamma^2 \gamma^3 \quad (14)$$

$$= -\gamma^2 \gamma^3 \gamma^2 \gamma^3 \quad (15)$$

$$= (\gamma^2)^2 (\gamma^3)^2 \quad (16)$$

$$= +1 \quad (17)$$

If the result after condensing contains zero, one, three or all four γ^ν s, it is equal to some constant multiplied by 1, γ^μ , $\gamma^5 \gamma^\mu$ or γ^5 , respectively. If the product contains two γ^ν s, we can see that it is a constant multiplied by a $\sigma^{\mu\nu}$, since

$$\sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu] \quad (18)$$

$$= \frac{i}{2} (2\gamma^\mu \gamma^\nu) \quad (19)$$

$$\gamma^\mu \gamma^\nu = -i\sigma^{\mu\nu} \quad (20)$$

Thus the product of any two of the Γ_r s is some linear combination of the members of the original set Γ .

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