DIRAC EQUATION: SET OF INDEPENDENT MATRICES

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References: Amitabha Lahiri & P. B. Pal, *A First Book of Quantum Field Theory*, Second Edition (Alpha Science International, 2004) - Chapter 4, Problem 4.2.

In the Dirac equation, we introduced 5 γ matrices labelled γ^μ and γ^5. These matrices satisfied the properties

$$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 \tag{1}$$
$$\{\gamma^\mu,\gamma^\nu\} = 2g^{\mu\nu} \tag{2}$$
$$\left(\gamma^0\right)^2 = 1 \tag{3}$$
$$\left(\gamma^i\right)^2 = -1 \tag{4}$$
$$\mathrm{Tr}\,\gamma^\mu = 0 \tag{5}$$
$$\mathrm{Tr}\,\gamma^5 = 0 \tag{6}$$
$$\left\{\gamma^\mu,\gamma^5\right\} = 0 \tag{7}$$
$$(\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0 \tag{8}$$

We also introduced the matrices

$$\sigma^{\mu\nu} \equiv \frac{i}{2}[\gamma^\mu,\gamma^\nu] = -\sigma^{\nu\mu} \tag{9}$$

We can form the set of 16 matrices defined as

$$\Gamma = \left\{1,\gamma^\mu,\sigma^{\mu\nu},\gamma^5\gamma^\mu,\gamma^5\right\} \tag{10}$$

We want to show that the product of any two Γ_rs is a linear combination of the existing Γ_rs.

The first thing to notice is that the elements of Γ as listed comprise matrices consisting of products of zero (that is, the unit matrix), one, two, three and four of the γ^μ. This is obvious for 1, γ^μ, $\sigma^{\mu\nu}$ and γ^5. To see that $\gamma^5\gamma^\mu$ is equivalent to the product of 3 γ^νs, use the anticommutation rules to swap γ^μ into γ^5 until it is adjacent to the γ^ν with $\nu=\mu$. We can now use 2 with

$\mu=\nu$ to see that $(\gamma^{\mu})^{2}=\pm1$, so the product $\gamma^{5}\gamma^{\mu}$ reduces to the product of the three remaining γ^{ν}s (times i).

In fact, we can use the same procedure to condense the product of any number of γ^{μ}s down to a product of 4 or fewer γ^{ν}s since, if there are more than 4 matrices in the product, some of them must be the same so by a process of swapping matrices using the anticommutation relations we can always bring equal matrices next to each other where they can be removed using $(\gamma^{\mu})^{2}=\pm1$.

In passing, we can also note that, from 1

$$\begin{aligned}
\left(\gamma^{5}\right)^{2} &= -\gamma^{0}\gamma^{1}\gamma^{2}\gamma^{3}\gamma^{0}\gamma^{1}\gamma^{2}\gamma^{3} && (11)\\
&= -(-1)^{3}\left(\gamma^{0}\right)^{2}\gamma^{1}\gamma^{2}\gamma^{3}\gamma^{1}\gamma^{2}\gamma^{3} && (12)\\
&= \gamma^{1}\gamma^{2}\gamma^{3}\gamma^{1}\gamma^{2}\gamma^{3} && (13)\\
&= (-1)^{2}\left(\gamma^{1}\right)^{2}\gamma^{2}\gamma^{3}\gamma^{2}\gamma^{3} && (14)\\
&= -\gamma^{2}\gamma^{3}\gamma^{2}\gamma^{3} && (15)\\
&= \left(\gamma^{2}\right)^{2}\left(\gamma^{3}\right)^{2} && (16)\\
&= +1 && (17)
\end{aligned}$$

If the result after condensing contains zero, one, three or all four γ^{ν}s, it is equal to some constant multiplied by 1, γ^{μ}, $\gamma^{5}\gamma^{\mu}$ or γ^{5}, respectively. If the product contains two γ^{ν}s, we can see that it is a constant multiplied by a $\sigma^{\mu\nu}$, since

$$\begin{aligned}
\sigma^{\mu\nu} &= \frac{i}{2}\left[\gamma^{\mu}\gamma^{\nu}-\gamma^{\nu}\gamma^{\mu}\right] && (18)\\
&= \frac{i}{2}\left(2\gamma^{\mu}\gamma^{\nu}\right) && (19)\\
\gamma^{\mu}\gamma^{\nu} &= -i\sigma^{\mu\nu} && (20)
\end{aligned}$$

Thus the product of any two of the Γ_{r}s is some linear combination of the members of the original set Γ.

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