

## DIRAC EQUATION: SET OF INDEPENDENT MATRICES

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References: Amitabha Lahiri & P. B. Pal, *A First Book of Quantum Field Theory*, Second Edition (Alpha Science International, 2004) - Chapter 4, Problem 4.2.

In the Dirac equation, we introduced 5  $\gamma$  matrices labelled  $\gamma^\mu$  and  $\gamma^5$ . These matrices satisfied the properties

$$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 \quad (1)$$

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \quad (2)$$

$$(\gamma^0)^2 = 1 \quad (3)$$

$$(\gamma^i)^2 = -1 \quad (4)$$

$$\text{Tr } \gamma^\mu = 0 \quad (5)$$

$$\text{Tr } \gamma^5 = 0 \quad (6)$$

$$\{\gamma^\mu, \gamma^5\} = 0 \quad (7)$$

$$(\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0 \quad (8)$$

We also introduced the matrices

$$\sigma^{\mu\nu} \equiv \frac{i}{2}[\gamma^\mu, \gamma^\nu] = -\sigma^{\nu\mu} \quad (9)$$

We can form the set of 16 matrices defined as

$$\Gamma = \left\{ 1, \gamma^\mu, \sigma^{\mu\nu}, \gamma^5\gamma^\mu, \gamma^5 \right\} \quad (10)$$

We want to show that the product of any two  $\Gamma_r$ s is a linear combination of the existing  $\Gamma_r$ s.

The first thing to notice is that the elements of  $\Gamma$  as listed comprise matrices consisting of products of zero (that is, the unit matrix), one, two, three and four of the  $\gamma^\mu$ . This is obvious for 1,  $\gamma^\mu$ ,  $\sigma^{\mu\nu}$  and  $\gamma^5$ . To see that  $\gamma^5\gamma^\mu$  is equivalent to the product of 3  $\gamma^\nu$ s, use the anticommutation rules to swap  $\gamma^\mu$  into  $\gamma^5$  until it is adjacent to the  $\gamma^\nu$  with  $\nu = \mu$ . We can now use 2 with

$\mu = \nu$  to see that  $(\gamma^\mu)^2 = \pm 1$ , so the product  $\gamma^5 \gamma^\mu$  reduces to the product of the three remaining  $\gamma^\nu$ s (times  $i$ ).

In fact, we can use the same procedure to condense the product of any number of  $\gamma^\mu$ s down to a product of 4 or fewer  $\gamma^\nu$ s since, if there are more than 4 matrices in the product, some of them must be the same so by a process of swapping matrices using the anticommutation relations we can always bring equal matrices next to each other where they can be removed using  $(\gamma^\mu)^2 = \pm 1$ .

In passing, we can also note that, from 1e

$$(\gamma^5)^2 = -\gamma^0 \gamma^1 \gamma^2 \gamma^3 \gamma^0 \gamma^1 \gamma^2 \gamma^3 \quad (11)$$

$$= -(-1)^3 (\gamma^0)^2 \gamma^1 \gamma^2 \gamma^3 \gamma^1 \gamma^2 \gamma^3 \quad (12)$$

$$= \gamma^1 \gamma^2 \gamma^3 \gamma^1 \gamma^2 \gamma^3 \quad (13)$$

$$= (-1)^2 (\gamma^1)^2 \gamma^2 \gamma^3 \gamma^2 \gamma^3 \quad (14)$$

$$= -\gamma^2 \gamma^3 \gamma^2 \gamma^3 \quad (15)$$

$$= (\gamma^2)^2 (\gamma^3)^2 \quad (16)$$

$$= +1 \quad (17)$$

If the result after condensing contains zero, one, three or all four  $\gamma^\nu$ s, it is equal to some constant multiplied by 1,  $\gamma^\mu$ ,  $\gamma^5 \gamma^\mu$  or  $\gamma^5$ , respectively. If the product contains two  $\gamma^\nu$ s, we can see that it is a constant multiplied by a  $\sigma^{\mu\nu}$ , since

$$\sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu] \quad (18)$$

$$= \frac{i}{2} (2\gamma^\mu \gamma^\nu) \quad (19)$$

$$\gamma^\mu \gamma^\nu = -i\sigma^{\mu\nu} \quad (20)$$

Thus the product of any two of the  $\Gamma_r$ s is some linear combination of the members of the original set  $\Gamma$ .

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