

GAMMA MATRICES: CONTRACTIONS

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References: Amitabha Lahiri & P. B. Pal, *A First Book of Quantum Field Theory*, Second Edition (Alpha Science International, 2004) - Chapter 4, Problem 4.7.

Here are a few formulas involving contractions of the gamma matrices. We make use of the anticommutation relation

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu} \quad (1)$$

and the squares

$$(\gamma_0)^2 = +1 \quad (2)$$

$$(\gamma_i)^2 = -1 \quad (3)$$

Note that for $i = 1, 2, 3$ (no sum over i):

$$\gamma_i \gamma^i = \gamma_i g^{ii} \gamma_i = (\gamma_i)^2 g^{ii} = +1 \quad (4)$$

The simple contraction is

$$\gamma_\lambda \gamma^\lambda = (\gamma_0)^2 + \sum_{i=1}^3 \gamma_i \gamma^i = 4 \quad (5)$$

Next, we have

$$\gamma_\lambda \gamma_\mu \gamma^\lambda = (2g_{\lambda\mu} - \gamma_\mu \gamma_\lambda) \gamma^\lambda \quad (6)$$

$$= 2\gamma_\mu - 4\gamma_\mu \quad (7)$$

$$= -2\gamma_\mu \quad (8)$$

Next:

$$\gamma_\lambda \gamma_\mu \gamma_\nu \gamma^\lambda = g^{\lambda\rho} \gamma_\lambda \gamma_\mu \gamma_\nu \gamma_\rho \quad (9)$$

$$= g^{\lambda\rho} (\gamma_\lambda \gamma_\mu (2g_{\nu\rho} - \gamma_\rho \gamma_\nu)) \quad (10)$$

$$= 2\gamma_\nu \gamma_\mu - (\gamma_\lambda \gamma_\mu \gamma^\lambda) \gamma_\nu \quad (11)$$

$$= 2\gamma_\nu \gamma_\mu + 2\gamma_\mu \gamma_\nu \quad (12)$$

$$= 2\gamma_\nu \gamma_\mu + 2(2g_{\mu\nu} - \gamma_\nu \gamma_\mu) \quad (13)$$

$$= 4g_{\mu\nu} \quad (14)$$

Finally:

$$\gamma_\lambda \gamma_\mu \gamma_\nu \gamma_\rho \gamma^\lambda = g^{\lambda\alpha} \gamma_\lambda \gamma_\mu \gamma_\nu \gamma_\rho \gamma_\alpha \quad (15)$$

$$= g^{\lambda\alpha} (\gamma_\lambda \gamma_\mu \gamma_\nu (2g_{\rho\alpha} - \gamma_\alpha \gamma_\rho)) \quad (16)$$

$$= 2\gamma_\rho \gamma_\mu \gamma_\nu - (\gamma_\lambda \gamma_\mu \gamma_\nu \gamma^\lambda) \gamma_\rho \quad (17)$$

$$= 2\gamma_\rho \gamma_\mu \gamma_\nu - 4g_{\mu\nu} \gamma_\rho \quad (18)$$

$$= 2\gamma_\rho (2g_{\mu\nu} - \gamma_\nu \gamma_\mu) - 4g_{\mu\nu} \gamma_\rho \quad (19)$$

$$= -2\gamma_\rho \gamma_\nu \gamma_\mu \quad (20)$$