

DIRAC SPINORS: GORDON IDENTITY

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References: Amitabha Lahiri & P. B. Pal, *A First Book of Quantum Field Theory*, Second Edition (Alpha Science International, 2004) - Chapter 4, Problem 4.9.

The Dirac spinors satisfy the relations

$$(\not{p} - m) u_s(\mathbf{p}) = 0 \quad (1)$$

$$(\not{p} + m) v_s(\mathbf{p}) = 0 \quad (2)$$

$$\bar{u}_s(\mathbf{p}) (\not{p} - m) = 0 \quad (3)$$

$$\bar{v}_s(\mathbf{p}) (\not{p} + m) = 0 \quad (4)$$

where a barred spinor is defined by

$$\bar{\psi} \equiv \psi^\dagger \gamma^0 \quad (5)$$

We've seen that these spinors satisfy the identities:

$$\bar{u}_s(\mathbf{p}) p^\mu u_r(\mathbf{p}) = m \bar{u}_s(\mathbf{p}) \gamma^\mu u_r(\mathbf{p}) \quad (6)$$

$$\bar{v}_s(\mathbf{p}) p^\mu v_r(\mathbf{p}) = -m \bar{v}_s(\mathbf{p}) \gamma^\mu v_r(\mathbf{p}) \quad (7)$$

The Gordon identity relates spinors at different momenta. We can derive it using the properties of the gamma matrices. To streamline the notation, I'll use the following definitions:

$$u \equiv u_s(\mathbf{p}) \quad (8)$$

$$u' \equiv u_s(\mathbf{p}') \quad (9)$$

with a similar notation for v and v' . The subscript s can be 1 or 2.

Multiply 1 on the left by $\bar{u}' \gamma^\mu$:

$$\bar{u}' \gamma^\mu \gamma^\alpha p_\alpha u = m \bar{u}' \gamma^\mu u \quad (10)$$

Now multiply 1 (with momentum \mathbf{p}') on the right by $\gamma^\mu u$:

$$\bar{u}' \gamma^\alpha p'_\alpha \gamma^\mu u = m \bar{u}' \gamma^\mu u \quad (11)$$

Now add these two equations:

$$\bar{u}' (\gamma^\mu \gamma^\alpha p_\alpha + \gamma^\alpha \gamma^\mu p'_\alpha) u = 2m \bar{u}' \gamma^\mu u \quad (12)$$

Using the anticommutator

$$\{\gamma^\mu, \gamma^\alpha\} = 2g^{\mu\alpha} \quad (13)$$

Now consider (using the definition of $\sigma^{\mu\alpha}$):

$$-i\sigma^{\mu\alpha} q_\alpha = -i\frac{i}{2} (\gamma^\mu \gamma^\alpha - \gamma^\alpha \gamma^\mu) (p_\alpha - p'_\alpha) \quad (14)$$

$$= \frac{1}{2} (\gamma^\mu \gamma^\alpha - 2g^{\alpha\mu} + \gamma^\mu \gamma^\alpha) p_\alpha + \quad (15)$$

$$\frac{1}{2} (\gamma^\alpha \gamma^\mu - 2g^{\mu\alpha} + \gamma^\alpha \gamma^\mu) p'_\alpha \quad (16)$$

$$= \gamma^\mu \gamma^\alpha p_\alpha + \gamma^\alpha \gamma^\mu p'_\alpha - (p + p')^\mu \quad (17)$$

The first two terms are the same as the quantity in parentheses in 12 so, substituting this back into 12 we have

$$\bar{u}' \gamma^\mu u = \frac{1}{2m} \bar{u}' \left[(p + p')^\mu - i\sigma^{\mu\alpha} q_\alpha \right] u \quad (18)$$

We can get a similar expression involving v by replacing m by $-m$:

$$\bar{v}' \gamma^\mu v = -\frac{1}{2m} \bar{v}' \left[(p + p')^\mu - i\sigma^{\mu\alpha} q_\alpha \right] v \quad (19)$$

This is the Gordon identity.