

DIRAC SPINORS: PRODUCTS OF SPINORS

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References: Amitabha Lahiri & P. B. Pal, *A First Book of Quantum Field Theory*, Second Edition (Alpha Science International, 2004) - Chapter 4, Problem 4.10.

The Dirac spinors satisfy the relations

$$(\not{p} - m) u_s(\mathbf{p}) = 0 \quad (1)$$

$$(\not{p} + m) v_s(\mathbf{p}) = 0 \quad (2)$$

$$\bar{u}_s(\mathbf{p})(\not{p} - m) = 0 \quad (3)$$

$$\bar{v}_s(\mathbf{p})(\not{p} + m) = 0 \quad (4)$$

where a barred spinor is defined by

$$\bar{\psi} \equiv \psi^\dagger \gamma^0 \quad (5)$$

Each spinor is a 4-component column vector, while the barred versions are each a 4-component row vector. Therefore, if we form the product $u_s \bar{u}_s$ we get a 4×4 matrix. If we sum over the two s indexes, we can prove the identities

$$\sum_s u_s(\mathbf{p}) \bar{u}_s(\mathbf{p}) = \not{p} + m \quad (6)$$

$$\sum_s v_s(\mathbf{p}) \bar{v}_s(\mathbf{p}) = \not{p} - m \quad (7)$$

We can show this by applying both sides of each equation to each of the four basis vectors u_s and v_s , where s can be $+$ or $-$ in each case. For the LHS, we can use the results

$$\bar{u}_s(\mathbf{p}) u_r(\mathbf{p}) = 2m \delta_{rs} \quad (8)$$

$$\bar{v}_s(\mathbf{p}) v_r(\mathbf{p}) = -2m \delta_{rs} \quad (9)$$

For example (all spinors are functions of \mathbf{p}):

$$\left[\sum_s u_s \bar{u}_s \right] u_- = u_+ \bar{u}_+ u_- + u_- \bar{u}_- u_- \quad (10)$$

$$= 0 + 2mu_- \quad (11)$$

$$= 2mu_- \quad (12)$$

The RHS is, using 1

$$(\not{p} + m) u_- = (\not{p} - m + 2m) u_- \quad (13)$$

$$= (\not{p} - m) u_- + 2mu_- \quad (14)$$

$$= 2mu_- \quad (15)$$

Thus $\sum_s \bar{u}_s(\mathbf{p}) u_s(\mathbf{p})$ has the same effect on u_- as $\not{p} + m$. The same argument works for u_+ .

For v_- , we can use the relation

$$\bar{u}_r(\mathbf{p}) v_s(\mathbf{p}) = 0 \quad (16)$$

Thus

$$\sum_s u_s \bar{u}_s v_s = 0 \quad (17)$$

and from 2

$$(\not{p} + m) v_s(\mathbf{p}) = 0 \quad (18)$$

Thus 6 is true when both sides are applied to all four basis vectors.

The argument for 7 is essentially the same. For example, for the LHS:

$$\left[\sum_s v_s \bar{v}_s \right] v_- = v_+ \bar{v}_+ v_- + v_- \bar{v}_- v_- \quad (19)$$

$$= 0 - 2mv_- \quad (20)$$

$$= -2mv_- \quad (21)$$

and for the RHS

$$(\not{p} - m) v_- = (\not{p} + m - 2m) v_- \quad (22)$$

$$= -2mv_- \quad (23)$$

Since both sides of both equations 6 and 7 have the same effect on all four basis vectors, the operators are equivalent.

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