

GAMMA MATRICES IN DIRAC-PAULI REPRESENTATION

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Post date: 14 June 2018.

References: Amitabha Lahiri & P. B. Pal, *A First Book of Quantum Field Theory*, Second Edition (Alpha Science International, 2004) - Chapter 4, Problem 4.13.

The gamma matrices that appear in the Dirac equation can, for the most part, be used without writing them down as explicit 4×4 matrices. To examine explicit solutions of the Dirac equation, however, it's useful to have a particular form for the matrices. One such form is the Dirac-Pauli representation, in which

$$\gamma^0 = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix} \quad (1)$$

$$\gamma^i = \begin{bmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{bmatrix} \quad (2)$$

where the σ^i are the Pauli matrices

$$\sigma^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}; \quad \sigma^2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}; \quad \sigma^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad (3)$$

Each of the entries in 1 and 2 is a 2×2 matrix, while the entries in the Pauli matrices are ordinary numbers. We can verify by direct calculation that this representation of the γ^μ s obey the required anticommutation relations and other relations, which we reproduce here for convenience:

$$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 \quad (4)$$

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \quad (5)$$

$$(\gamma^0)^2 = 1 \quad (6)$$

$$(\gamma^i)^2 = -1 \quad (7)$$

$$\text{Tr } \gamma^\mu = 0 \quad (8)$$

$$\text{Tr } \gamma^5 = 0 \quad (9)$$

$$\{\gamma^\mu, \gamma^5\} = 0 \quad (10)$$

$$(\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0 \quad (11)$$

We can work out γ^5 in this representation by direct multiplication.

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 \quad (12)$$

$$= i \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix} \begin{bmatrix} 0 & \sigma^1 \\ -\sigma^1 & 0 \end{bmatrix} \begin{bmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{bmatrix} \begin{bmatrix} 0 & \sigma^3 \\ -\sigma^3 & 0 \end{bmatrix} \quad (13)$$

$$= i \begin{bmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{bmatrix} \begin{bmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{bmatrix} \begin{bmatrix} 0 & \sigma^3 \\ -\sigma^3 & 0 \end{bmatrix} \quad (14)$$

$$= i \begin{bmatrix} -\sigma^1\sigma^2 & 0 \\ 0 & \sigma^1\sigma^2 \end{bmatrix} \begin{bmatrix} 0 & \sigma^3 \\ -\sigma^3 & 0 \end{bmatrix} \quad (15)$$

$$= i \begin{bmatrix} 0 & -\sigma^1\sigma^2\sigma^3 \\ -\sigma^1\sigma^2\sigma^3 & 0 \end{bmatrix} \quad (16)$$

By multiplying out 3 we find

$$\sigma^1\sigma^2\sigma^3 = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} = iI \quad (17)$$

so we have

$$\gamma^5 = i \begin{bmatrix} 0 & -iI \\ -iI & 0 \end{bmatrix} = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \quad (18)$$

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