GAMMA MATRICES IN DIRAC-PAULI REPRESENTATION

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References: Amitabha Lahiri & P. B. Pal, *A First Book of Quantum Field Theory*, Second Edition (Alpha Science International, 2004) - Chapter 4, Problem 4.13.

The gamma matrices that appear in the Dirac equation can, for the most part, be used without writing them down as explicit 4×4 matrices. To examine explicit solutions of the Dirac equation, however, it's useful to have a particular form for the matrices. One such form is the Dirac-Pauli representation, in which

$$\gamma^0 = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix} \tag{1}$$

$$\gamma^i = \begin{bmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{bmatrix} \tag{2}$$

where the σ^i are the Pauli matrices

$$\sigma^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}; \quad \sigma^2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}; \quad \sigma^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{3}$$

Each of the entries in 1 and 2 is a 2×2 matrix, while the entries in the Pauli matrices are ordinary numbers. We can verify by direct calculation that this representation of the γ^μs obey the required anticommutation relations and other relations, which we reproduce here for convenience:

$$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 \tag{4}$$
$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \tag{5}$$
$$\left(\gamma^0\right)^2 = 1 \tag{6}$$
$$\left(\gamma^i\right)^2 = -1 \tag{7}$$
$$\text{Tr } \gamma^\mu = 0 \tag{8}$$
$$\text{Tr } \gamma^5 = 0 \tag{9}$$
$$\left\{\gamma^\mu, \gamma^5\right\} = 0 \tag{10}$$
$$(\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0 \tag{11}$$

We can work out γ^5 in this representation by direct multiplication.

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 \tag{12}$$
$$= i\begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}\begin{bmatrix} 0 & \sigma^1 \\ -\sigma^1 & 0 \end{bmatrix}\begin{bmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{bmatrix}\begin{bmatrix} 0 & \sigma^3 \\ -\sigma^3 & 0 \end{bmatrix} \tag{13}$$
$$= i\begin{bmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{bmatrix}\begin{bmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{bmatrix}\begin{bmatrix} 0 & \sigma^3 \\ -\sigma^3 & 0 \end{bmatrix} \tag{14}$$
$$= i\begin{bmatrix} -\sigma^1\sigma^2 & 0 \\ 0 & \sigma^1\sigma^2 \end{bmatrix}\begin{bmatrix} 0 & \sigma^3 \\ -\sigma^3 & 0 \end{bmatrix} \tag{15}$$
$$= i\begin{bmatrix} 0 & -\sigma^1\sigma^2\sigma^3 \\ -\sigma^1\sigma^2\sigma^3 & 0 \end{bmatrix} \tag{16}$$

By multiplying out 3 we find

$$\sigma^1\sigma^2\sigma^3 = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} = iI \tag{17}$$

so we have

$$\gamma^5 = i\begin{bmatrix} 0 & -iI \\ -iI & 0 \end{bmatrix} = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \tag{18}$$

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