

PROPAGATOR FOR THE DIRAC FIELD

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

Post date: 29 June 2018.

References: Amitabha Lahiri & P. B. Pal, *A First Book of Quantum Field Theory*, Second Edition (Alpha Science International, 2004) - Chapter 4, Problem 4.29.

The propagator for the Dirac field is derived using similar logic to that for the scalar field. The Dirac equation is coupled to a source term:

$$(i\gamma^\mu \partial_\mu - m) \psi(x) = J(x) \quad (1)$$

Note that $J(x)$ is not angular momentum in this context.

The Dirac propagator S is defined by the equation

$$(i\gamma^\mu \partial_\mu - m) S(x-x') = \delta^4(x-x') \quad (2)$$

so the general solution of the Dirac equation is then

$$\psi(x) = \psi_0(x) + \int d^4x' S(x-x') J(x') \quad (3)$$

The crucial difference between the Dirac case and the scalar field is that here, ψ and J are 4-component vectors and S is a 4×4 matrix. Thus the RHS of 2 is implicitly multiplied by the unit matrix I .

Taking a Fourier transform we get

$$S(x-x') = \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot (x-x')} S(p) \quad (4)$$

From 2, we have

$$(i\cancel{\partial} - m) S(x-x') = \int \frac{d^4p}{(2\pi)^4} (i(-i\cancel{p}) - m) e^{-ip \cdot (x-x')} S(p) \quad (5)$$

$$= \int \frac{d^4p}{(2\pi)^4} (\cancel{p} - m) e^{-ip \cdot (x-x')} S(p) \quad (6)$$

Since

$$\delta^4(x-x') = \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot (x-x')} \quad (7)$$

if we require the RHS of 6 to be $\delta^4(x-x')$ we must have

$$(\not{p} - m) S(p) = 1 \quad (8)$$

Multiplying both sides by $\not{p} + m$ we have

$$(p^2 - m^2) S(p) = \not{p} + m \quad (9)$$

$$S(p) = \frac{\not{p} + m}{p^2 - m^2} \quad (10)$$

When doing the integral over p^0 in 6, this denominator has poles at $p^0 = \pm E_p = \pm \sqrt{\mathbf{p}^2 + m^2}$, just as in the scalar field case, so we use the same fix here by introducing an infinitesimal imaginary part to the denominator:

$$S_F(p) = \frac{\not{p} + m}{p^2 - m^2 + i\epsilon} \quad (11)$$

S_F is called the Feynman propagator. With this alteration, 4 becomes

$$S_F(x - x') = \int \frac{d^4 p}{(2\pi)^4} e^{-ip \cdot (x - x')} \frac{\not{p} + m}{p^2 - m^2 + i\epsilon} \quad (12)$$

Using the propagator for scalar fields that we found earlier:

$$\Delta_F(x - x') = \int \frac{d^4 p}{(2\pi)^4} \frac{e^{-ip \cdot (x - x')}}{p^2 - m^2 + i\epsilon} \quad (13)$$

we can write 12 as

$$S_F(x - x') = (i\not{\partial} + m) \Delta_F(x - x') \quad (14)$$

We can use the result, obtained by doing the integral over p^0 , given in L&P's eqn 3.76, which is:

$$i\Delta_F(x - x') = \int \frac{d^3 p}{(2\pi)^3 2E_p} \left[\Theta(t - t') e^{-ip \cdot (x - x')} + \Theta(t' - t) e^{ip \cdot (x - x')} \right] \quad (15)$$

Applying the operator $i\not{\partial} + m$ to this gives us

$$iS_F(x-x') = \int \frac{d^3p}{(2\pi)^3 2E_p} \left[\Theta(t-t') (\not{p} + m) e^{-ip \cdot (x-x')} + \right. \quad (16)$$

$$\left. \Theta(t'-t) (-\not{p} + m) e^{ip \cdot (x-x')} \right] \quad (17)$$

$$= \int \frac{d^3p}{(2\pi)^3 2E_p} \left[\Theta(t-t') (\not{p} + m) e^{-ip \cdot (x-x')} - \right. \quad (18)$$

$$\left. \Theta(t'-t) (\not{p} - m) e^{ip \cdot (x-x')} \right] \quad (19)$$

Here $\Theta(t-t')$ is the step function which is zero for $t < t'$ and 1 for $t > t'$.

PINGBACKS

Pingback: Propagator for the Dirac field as a time-ordered product