PROPAGATOR FOR THE DIRAC FIELD

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References: Amitabha Lahiri & P. B. Pal, *A First Book of Quantum Field Theory*, Second Edition (Alpha Science International, 2004) - Chapter 4, Problem 4.29.

The propagator for the Dirac field is derived using similar logic to that for the scalar field. The Dirac equation is coupled to a source term:

$$(i\gamma^{\mu}\partial_{\mu}-m)\psi(x)=J(x) \tag{1}$$

Note that $J(x)$ is not angular momentum in this context.

The Dirac propagator S is defined by the equation

$$(i\gamma^{\mu}\partial_{\mu}-m)S\left(x-x'\right)=\delta^{4}\left(x-x'\right) \tag{2}$$

so the general solution of the Dirac equation is then

$$\psi(x)=\psi_{0}(x)+\int d^{4}x\;S\left(x-x'\right)J\left(x'\right) \tag{3}$$

The crucial difference between the Dirac case and the scalar field is that here, ψ and J are 4-component vectors and S is a 4×4 matrix. Thus the RHS of 2 is implicity multiplied by the unit matrix I.

Taking a Fourier transform we get

$$S\left(x-x'\right)=\int\frac{d^{4}p}{(2\pi)^{4}}e^{-ip\cdot(x-x')}S(p) \tag{4}$$

From 2, we have

$$\left(i\partial\!\!\!/-m\right)S\left(x-x'\right)=\int\frac{d^{4}p}{(2\pi)^{4}}\left(i\left(-ip\!\!\!/\right)-m\right)e^{-ip\cdot(x-x')}S(p) \tag{5}$$

$$=\int\frac{d^{4}p}{(2\pi)^{4}}\left(p\!\!\!/-m\right)e^{-ip\cdot(x-x')}S(p) \tag{6}$$

Since

$$\delta^{4}\left(x-x'\right)=\int\frac{d^{4}p}{(2\pi)^{4}}e^{-ip\cdot(x-x')} \tag{7}$$

if we require the RHS of 6 to be $\delta^{4}(x-x')$ we must have

$$\left(\not{p}-m\right)S\left(p\right)=1 \tag{8}$$

Multiplying both sides by $\not{p}+m$ we have

$$\left(p^2-m^2\right)S\left(p\right)=\not{p}+m \tag{9}$$

$$S\left(p\right)=\frac{\not{p}+m}{p^2-m^2} \tag{10}$$

When doing the integral over p^0 in 6, this denominator has poles at $p^0=\pm E_p=\pm\sqrt{\mathbf{p}^2+m^2}$, just as in the scalar field case, so we use the same fix here by introducing an infinitesimal imaginary part to the denominator:

$$S_F\left(p\right)=\frac{\not{p}+m}{p^2-m^2+i\varepsilon} \tag{11}$$

S_F is called the Feynman propagator. With this alteration, 4 becomes

$$S_F\left(x-x'\right)=\int\frac{d^4p}{(2\pi)^4}e^{-ip\cdot(x-x')}\frac{\not{p}+m}{p^2-m^2+i\varepsilon} \tag{12}$$

Using the propagator for scalar fields that we found earlier:

$$\Delta_F\left(x-x'\right)=\int\frac{d^4p}{(2\pi)^4}\frac{e^{-ip\cdot(x-x')}}{p^2-m^2+i\varepsilon} \tag{13}$$

we can write 12 as

$$S_F\left(x-x'\right)=\left(i\not{\partial}+m\right)\Delta_F\left(x-x'\right) \tag{14}$$

We can use the result, obtained by doing the integral over p^0, given in L&P's eqn 3.76, which is:

$$i\Delta_F\left(x-x'\right)=\int\frac{d^3p}{(2\pi)^3\,2E_p}\left[\Theta\left(t-t'\right)e^{-ip\cdot(x-x')}+\Theta\left(t'-t\right)e^{ip\cdot(x-x')}\right] \tag{15}$$

Applying the operator $i\not{\partial}+m$ to this gives us

$$iS_F\left(x-x'\right)=\int\frac{d^3p}{(2\pi)^3\,2E_p}\left[\Theta\left(t-t'\right)\left(\not p+m\right)e^{-ip\cdot(x-x')}+\right. \tag{16}$$

$$\left.\Theta\left(t'-t\right)\left(-\not p+m\right)e^{ip\cdot(x-x')}\right] \tag{17}$$

$$=\int\frac{d^3p}{(2\pi)^3\,2E_p}\left[\Theta\left(t-t'\right)\left(\not p+m\right)e^{-ip\cdot(x-x')}-\right. \tag{18}$$

$$\left.\Theta\left(t'-t\right)\left(\not p-m\right)e^{ip\cdot(x-x')}\right] \tag{19}$$

Here $\Theta\left(t-t'\right)$ is the step function which is zero for $t<t'$ and 1 for $t>t'$.

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