WICK'S THEOREM - NO EQUAL TIME CONTRACTIONS

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References: Amitabha Lahiri & P. B. Pal, *A First Book of Quantum Field Theory*, Second Edition (Alpha Science International, 2004) - Chapter 5.

Wick's theorem states that we can express a time-ordered product of operators as a series of terms involving normal ordered products and contractions, with the result

$$\mathscr{T}\left[ABC\ldots XYZ\right] = :ABC\ldots XYZ: + \tag{1}$$

$$:\underbrace{AB}C\ldots XYZ: + \tag{2}$$

$$:\underbrace{ABC}\ldots XYZ: + \tag{3}$$

$$:ABC\ldots X\underbrace{YZ}: + \tag{4}$$

$$:\underbrace{AB}\,\underbrace{CD}\ldots XYZ: + \tag{5}$$

$$:\wick{\c1 A \c2 B \c1 C \c2 D}\ldots XYZ: \tag{6}$$

$$:ABC\ldots \underbrace{WX}\,\underbrace{YZ}: + \tag{7}$$

$$\text{all higher order contractions} \tag{8}$$

This theorem is to be applied to the calculation of the S-matrix which is defined as the limit at infinite time of the evolution operator

$$S \equiv \lim_{t\to\infty} U(t) = 1 + \sum_{n=1}^{\infty} \frac{(-i)^n}{n!} \int d^4x_1 \int d^4x_2 \ldots \int d^4x_n \tag{9}$$

$$\mathscr{T}\left[\mathscr{H}_I(x_1)\,\mathscr{H}_I(x_2)\ldots\mathscr{H}_I(x_n)\right] \tag{10}$$

where we've written the integrals as being done over the hamiltonian densities, so all integrals are over all four spacetime coordinates.

In practice, each of the interaction hamiltonian terms $\mathscr{H}_I(x)$ in the time-ordered product will usually be a product of several field operators such as ψ and $\overline{\psi}$ for fermions and ϕ for scalars. Since each H_I is evaluated at a specific time, the time arguments of all its constituent fields will be the same. As a result, if for example

$$\mathscr{H}_I(x) = A(x)\,B(x)\,C(x) \tag{11}$$

we're faced with a time-ordered product of the form

$$\mathscr{T}\left[\mathscr{H}_I(x_1)\mathscr{H}_I(x_2)\ldots\mathscr{H}_I(x_n)\right]=\mathscr{T}\left[:ABC(x_1):\,:ABC(x_2)\ldots:ABC(x_n)::\right] \tag{12}$$

As all field operators within each normal-ordered product are at the same spacetime point, their commutators or anticommutators can be infinite, which we need to avoid. To avoid this, an infinitesimal time ϵ_r is added to each creation operator and subtracted from each annihilation operator within the fields in each normal-ordered product. This has the effect of placing all the creation operators to the left of all annihilation operators when time ordering is performed. That ordering is automatically normal-ordered as well, so we don't need to apply Wick's theorem to the fields within each normal-ordered product, as they are already time-ordered. The net result is that, whenever we are faced with a time-ordered product containing fields that are at equal times, we don't do any contractions over pairs of such fields. As stated by L&P, this is given by

$$\mathscr{T}\left[:ABC(x_1):\,:ABC(x_2):\ldots:ABC(x_n):\right]=\mathscr{T}\left[(ABC(x_1))\ (ABC(x_2))\ldots(ABC(x_n))\right]_{\text{no e.t.c.}} \tag{13}$$

where "no e.t.c." means "no equal-time contractions".

This again seems like a bit of a kludge (along the lines of normal ordering, which seemed quite kludgy in the first place) but I suppose it is justified by the fact that the results do seem to agree with experiments.

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