

WICK'S THEOREM - NO EQUAL TIME CONTRACTIONS

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References: Amitabha Lahiri & P. B. Pal, *A First Book of Quantum Field Theory*, Second Edition (Alpha Science International, 2004) - Chapter 5.

Wick's theorem states that we can express a time-ordered product of operators as a series of terms involving normal ordered products and contractions, with the result

$$\mathcal{T}[ABC\dots XYZ] = :ABC\dots XYZ: + \quad (1)$$

$$:\underbrace{ABC\dots XYZ}: + \quad (2)$$

$$:\underbrace{ABC\dots XYZ}: + \quad (3)$$

$$:\underbrace{ABC\dots XYZ}: + \quad (4)$$

$$:\underbrace{ABC}\underbrace{D\dots XYZ}: + \quad (5)$$

$$:\underbrace{ABC}\underbrace{D\dots XYZ}: \quad (6)$$

$$:\underbrace{ABC\dots W}\underbrace{XYZ}: + \quad (7)$$

$$\text{all higher order contractions} \quad (8)$$

This theorem is to be applied to the calculation of the S-matrix which is defined as the limit at infinite time of the evolution operator

$$S \equiv \lim_{t \rightarrow \infty} U(t) = 1 + \sum_{n=1}^{\infty} \frac{(-i)^n}{n!} \int d^4x_1 \int d^4x_2 \dots \int d^4x_n \quad (9)$$

$$\mathcal{T}[\mathcal{H}_I(x_1)\mathcal{H}_I(x_2)\dots\mathcal{H}_I(x_n)] \quad (10)$$

where we've written the integrals as being done over the hamiltonian densities, so all integrals are over all four spacetime coordinates.

In practice, each of the interaction hamiltonian terms $\mathcal{H}_I(x)$ in the time-ordered product will usually be a product of several field operators such as ψ and $\bar{\psi}$ for fermions and ϕ for scalars. Since each H_I is evaluated at a specific time, the time arguments of all its constituent fields will be the same. As a result, if for example

$$\mathcal{H}_I(x) = A(x)B(x)C(x) \quad (11)$$

we're faced with a time-ordered product of the form

$$\mathcal{T} [\mathcal{H}_I(x_1) \mathcal{H}_I(x_2) \dots \mathcal{H}_I(x_n)] = \mathcal{T} [:ABC(x_1): :ABC(x_2) \dots :ABC(x_n):] \quad (12)$$

As all field operators within each normal-ordered product are at the same spacetime point, their commutators or anticommutators can be infinite, which we need to avoid. To avoid this, an infinitesimal time ϵ_r is added to each creation operator and subtracted from each annihilation operator within the fields in each normal-ordered product. This has the effect of placing all the creation operators to the left of all annihilation operators when time ordering is performed. That ordering is automatically normal-ordered as well, so we don't need to apply Wick's theorem to the fields within each normal-ordered product, as they are already time-ordered. The net result is that, whenever we are faced with a time-ordered product containing fields that are at equal times, we don't do any contractions over pairs of such fields. As stated by L&P, this is given by

$$\mathcal{T} [:ABC(x_1): :ABC(x_2): \dots :ABC(x_n):] = \mathcal{T} [(ABC(x_1)) (ABC(x_2)) \dots (ABC(x_n))]_{\text{no e.t.c.}} \quad (13)$$

where "no e.t.c." means "no equal-time contractions".

This again seems like a bit of a kludge (along the lines of normal ordering, which seemed quite kludgy in the first place) but I suppose it is justified by the fact that the results do seem to agree with experiments.

PINGBACKS

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