

S-MATRIX

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References: Amitabha Lahiri & P. B. Pal, *A First Book of Quantum Field Theory*, Second Edition (Alpha Science International, 2004) - Chapter 5.

The evolution operator $U(t)$ is used to follow the evolution of an initial state through an interaction and out the other side into a final state. It is written as a series over successively higher-order integrals:

$$U(t) = 1 + \sum_{n=1}^{\infty} \frac{(-i)^n}{n!} \int_{-\infty}^t dt_1 \int_{-\infty}^{t_1} dt_2 \dots \int_{-\infty}^{t_{n-1}} dt_n \quad (1)$$

$$\mathcal{T} [H_I(t_1) H_I(t_2) \dots H_I(t_n)] \quad (2)$$

where $H_I(t)$ is the interaction hamiltonian at time t . This is often written in the abbreviated form

$$U(t) = \mathcal{T} \left[\exp \left(-i \int_{-\infty}^t dt' H_I(t') \right) \right] \quad (3)$$

where the exponential is taken to represent the series in the first equation.

The S -matrix is defined as the limit of U at infinite upper time, that is

$$S \equiv \lim_{t \rightarrow \infty} U(t) = \mathcal{T} \left[\exp \left(-i \int_{-\infty}^{\infty} dt' H_I(t') \right) \right] \quad (4)$$

Since H_I is the total hamiltonian, it can be written as the integral of the hamiltonian density $\mathcal{H}_I(x)$ where x is now the spacetime four-vector. That is

$$H_I = \int d^3x \mathcal{H}_I(x) \quad (5)$$

If we combine the space integral with the time integral, we can write 4 as

$$S = \mathcal{T} \left[\exp \left(-i \int d^4x \mathcal{H}_I(x) \right) \right] \quad (6)$$

where now all four coordinates are integrated over all values. Since U is unitary, so is S :

$$S^\dagger = S^{-1} \quad (7)$$

$$S^\dagger S = 1 \quad (8)$$

The value of the S -matrix is that its matrix elements represent the amplitudes for the processes in which we start with some initial state $|i\rangle$ to a final state $|f\rangle$. In other words, if we know the S -matrix, we have a prediction for the probabilities of the various outcomes of particle experiments involving decay and scattering, so the S -matrix is central to the predictive ability of quantum field theory. In symbols, L&P show leading up to their eqn 5.39, that the amplitude for making a transition from the initial state $|i\rangle$ to the final state $|f\rangle$ is $\langle f|S|i\rangle$.

The catch, of course, is that it's far from easy to calculate S , since it involves calculating an infinite series of nested integrals, as in 2. In practice, we must specify the interaction hamiltonian (or, more usually, we begin with a Lagrangian and derive the hamiltonian from that), and then hope that the successive terms in 2 diminish fast enough that we can get a meaningful prediction using just the first few terms in the series. To calculate these terms, we note that hamiltonians are usually given as normal-ordered products, while the terms in 2 are time ordered, so we need a way of relating a time-ordered product to a normal-ordered product. This is given by Wick's theorem.

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