DECAY OF A MOVING BOSON INTO A FERMION-ANTIFERMION PAIR

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References: Amitabha Lahiri & P. B. Pal, *A First Book of Quantum Field Theory*, Second Edition (Alpha Science International, 2004) - Chapter 7, Exercise 7.3.

In their section 7.2, L&P calculate the decay rate Γ of a boson into a fermion-antifermion pair in the rest frame of the boson, at the lowest order in the Yukawa interaction. In this problem, we examine the same process but take the boson to be moving in the z direction with velocity v, so that its four-momentum is

$$k = \gamma M(1,0,0,v) \tag{1}$$

where M is the rest mass of the boson and γ is the usual relativistic factor

$$\gamma = \frac{1}{\sqrt{1-v^2}} \tag{2}$$

To modify L&P's calculation to take account of the moving boson, we can follow through their derivation and note where the changes are needed. They begin with their equation 7.8, which gives the decay rate in general.

$$\Gamma = \frac{1}{2E_i}\int\prod_f\left[\frac{d^3p}{(2\pi)^3 2E}\right]_f (2\pi)^4\delta^4\left(p_i - \sum_f p_f\right)\left|\mathscr{M}_{fi}\right|^2 \tag{3}$$

Here E_i is the energy of the initial particle (the boson in this case) and the product $\prod_f$ is taken over all final particles. The notation can be a bit confusing as it's difficult to tell which index f's are included in the product. Only the term immediately following the $\prod_f$ (in square brackets) is included in the product. The term $\mathscr{M}_{fi}$ is the Feynman amplitude for the overall process, so is not part of the product.

In our case, the energy of the boson is

$$E_i = \gamma M \tag{4}$$

and we have two outgoing particles, so 3 is

$$\Gamma = \frac{1}{2\gamma M}\int \frac{d^3p}{(2\pi)^3 2E}\int \frac{d^3p'}{(2\pi)^3 2E'}(2\pi)^4\delta^4\left(k-p-p'\right)\left|\mathscr{M}_{fi}\right|^2 \tag{5}$$

where k is the four-momentum of the incoming boson, and p and p' are the momenta of the outgoing fermion and antifermion. The only difference between this formula and L&P's equation 7.9 is that there is an extra factor of γ in the denominator.

The calculation of the Feynman amplitude for the case of a moving boson is unaltered from L&P's treatment over their equations 7.10 to 7.27 so we refer the reader to their book for the details. We end up with

$$\Gamma = \frac{h^2\left(M^2-4m^2\right)}{\gamma M}\rho \tag{6}$$

where h is the interaction constant in the Yukawa interaction, m is the mass of the fermion (or antifermion) and

$$\rho = \int \frac{d^3p}{(2\pi)^3 2E}\int \frac{d^3p'}{(2\pi)^3 2E'}(2\pi)^4\delta^4\left(k-p-p'\right) \tag{7}$$

At this point, we can observe that, due to momentum conservation, the sum of the outgoing fermion and antifermion momenta must equal the momentum of the incoming boson. Since the fermion and antifermion have the same mass and the incoming boson momentum is entirely in the z direction, the x and y components of the outgoing momenta must exactly cancel, and the z components of the two particles must each be equal to half the boson's momentum. This means that the magnitudes of the two momenta are equal, and this implies that the energies of the fermion and antifermion must be equal, so $E = E'$. As in L&P's equation 7.29, we can do the integral over the spatial components of p' to get

$$\rho = \int \frac{d^3p}{(2\pi)^2 4E^2}\delta\left(k_0-p_0-p'_0\right) \tag{8}$$

The components in the delta function have the values

$$k_0 = \gamma M \tag{9}$$

$$p_0 = p'_0 = E \tag{10}$$

so we have

$$\rho = \int \frac{d^3p}{(2\pi)^2 4E^2}\delta\left(\gamma M - 2E\right) \tag{11}$$

Using the property of the delta function

$$\delta(kx) = \frac{1}{|k|}\delta(x) \tag{12}$$

we have (since E is the variable):

$$\delta(\gamma M - 2E) = \frac{1}{2}\delta\left(E - \frac{\gamma M}{2}\right) \tag{13}$$

In 11, $E = \sqrt{\mathfrak{p}^2 + m^2}$ (where $\mathfrak{p} \equiv |\mathbf{p}|$) so does not depend on the angular direction of $\mathbf{p}$, so we can integrate over these angles to get

$$\rho = \frac{1}{\pi}\int \frac{d\mathfrak{p}\ \mathfrak{p}^2}{4E^2}\frac{1}{2}\delta\left(E - \frac{\gamma M}{2}\right) \tag{14}$$

However, in the problem statement, it says we can neglect the mass m of the electron which makes $E = \mathfrak{p}$, so the integral is just

$$\rho \approx \frac{1}{8\pi}\int d\mathfrak{p}\ \delta\left(\mathfrak{p} - \frac{\gamma M}{2}\right) = \frac{1}{8\pi} \tag{15}$$

Plugging this into 6 and taking $m = 0$ we have

$$\Gamma = \frac{Mh^2}{8\pi\gamma} \tag{16}$$

Within this approximation, we see that Γ for the moving boson is reduced by a factor of $\gamma \geq 1$ from the same quantity for the boson at rest. In other words, the moving boson appears to decay more slowly than the boson at rest, which is what we would expect due to time dilation. This was one of results that confirmed the theory of special relativity, where muons cascading through the Earth's atmosphere were observed to decay more slowly than muons at rest, resulting in more of them reaching the ground.

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