BOSON DECAY WITH MODIFIED YUKAWA INTERACTION

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In their section 7.2, L&P calculate the decay rate Γ of a boson into a fermion-antifermion pair in the rest frame of the boson, at the lowest order in the Yukawa interaction. We are then asked to calculate this decay rate using the interaction Lagrangian

$$\mathscr{L}_{\text{int}} = -h'\overline{\psi}\gamma_5\psi\phi \tag{1}$$

We've seen that the interaction constant h' must be purely imaginary in order that $\mathscr{L}_{\text{int}}$ is hermitian.

If we trace through L&P's derivation in section 7.2 for the Yukawa interaction, we see that the only place where the explicit form of the Lagrangian is used is in the calculation of the Feynman amplitude, which for our case when we compare with L&P's equation 6.50, becomes (in the lowest order)

$$i\mathscr{M}_{fi} = \left(-ih'\right)\overline{u}_s(\mathbf{p})\gamma_5 v_{s'}\left(\mathbf{p}'\right) \tag{2}$$

Inserting this into L&P's equation 7.9 we get, after summing over spins

$$\Gamma = \frac{|h'|^2}{2M}\int\frac{d^3p}{(2\pi)^3 2E}\int\frac{d^3p'}{(2\pi)^3 2E'}(2\pi)^4\delta^4\left(k-p-p'\right)\sum_{s,s'}\left|\overline{u}_s(\mathbf{p})\gamma_5 v_{s'}\left(\mathbf{p}'\right)\right|^2 \tag{3}$$

To evaluate the spin sum, we can use the result for an arbitrary 4×4 matrix F:

$$\sum_{s,s'}\left|\overline{u}_s(\mathbf{p})Fv_{s'}\left(\mathbf{p}'\right)\right|^2 = \text{Tr}\left[\left(\not{p}+m\right)F\left(\not{p}'-m\right)F^{\ddagger}\right] \tag{4}$$

where

$$F^{\ddagger} \equiv \gamma_0 F^{\dagger}\gamma_0 \tag{5}$$

In our case, $F=\gamma_5$ so we have

$$\sum_{s,s'}\left|\overline{u}_s(\mathbf{p})Fv_{s'}\left(\mathbf{p}'\right)\right|^2 = \text{Tr}\left[\left(\not{p}+m\right)\gamma_5\left(\not{p}'-m\right)\gamma_0\gamma_5^{\dagger}\gamma_0\right]$$

To work out the trace, we can use some properties of the gamma matrices, in particular

$$\mathrm{Tr}\gamma^{\mu} = \mathrm{Tr}\gamma_5 = 0 \tag{6}$$
$$(\gamma_0)^2 = (\gamma_5)^2 = I \tag{7}$$
$$\{\gamma^{\mu}, \gamma_5\} = 0 \tag{8}$$
$$\{\gamma^{\mu}, \gamma^{\nu}\} = 2g^{\mu\nu} \tag{9}$$
$$\not{p} \equiv \gamma^{\mu} p_{\mu} \tag{10}$$

Our sum is thus

$$\sum_{s,s'} \left|\overline{u}_s(\mathbf{p}) F v_{s'}(\mathbf{p}')\right|^2 = \mathrm{Tr}\left[(\not{p} + m)\gamma_5(\not{p}' - m)\gamma_0\gamma_5\gamma_0\right] \tag{11}$$

There are 4 terms in 11 to evaluate. First

$$\begin{aligned}
\mathrm{Tr}\left(\not{p}\gamma_5\not{p}'\gamma_0\gamma_5\gamma_0\right) &= p^{\mu}p'^{\nu}\mathrm{Tr}\left(\gamma_{\mu}\gamma_5\gamma_{\nu}\gamma_0\gamma_5\gamma_0\right) &(12)\\
&= -p^{\mu}p'^{\nu}\mathrm{Tr}\left(\gamma_{\mu}\gamma_5\gamma_{\nu}(\gamma_0)^2\gamma_5\right) &(13)\\
&= -p^{\mu}p'^{\nu}\mathrm{Tr}\left(\gamma_{\mu}\gamma_5\gamma_{\nu}\gamma_5\right) &(14)\\
&= p^{\mu}p'^{\nu}\mathrm{Tr}\left(\gamma_{\mu}\gamma_{\nu}(\gamma_5)^2\right) &(15)\\
&= p^{\mu}p'^{\nu}\mathrm{Tr}\left(\gamma_{\mu}\gamma_{\nu}\right) &(16)\\
&= \frac{1}{2}p^{\mu}p'^{\nu}\mathrm{Tr}\left(\gamma_{\mu}\gamma_{\nu} + \gamma_{\nu}\gamma_{\mu}\right) &(17)\\
&= p^{\mu}p'^{\nu}\mathrm{Tr}\left(g_{\mu\nu}\right) &(18)\\
&= p \cdot p'\mathrm{Tr}I &(19)\\
&= 4p \cdot p' &(20)
\end{aligned}$$

where we've used 9.

Now consider

$$\begin{aligned}
\mathrm{Tr}\left(\not{p}\gamma_5 m\gamma_0\gamma_5\gamma_0\right) &= p^{\mu}\mathrm{Tr}\left(\gamma_{\mu}\gamma_5 m\gamma_0\gamma_5\gamma_0\right) &(21)\\
&= -mp^{\mu}\mathrm{Tr}\left(\gamma_{\mu}\gamma_5(\gamma_0)^2\gamma_5\right) &(22)\\
&= -mp^{\mu}\mathrm{Tr}\left(\gamma_{\mu}(\gamma_5)^2\right) &(23)\\
&= -mp^{\mu}\mathrm{Tr}\gamma_{\mu} &(24)\\
&= 0 &(25)
\end{aligned}$$

Similarly

$$\text{Tr}\left(m\gamma_5 p\!\!\!/'\gamma_0\gamma_5\gamma_0\right)=0 \tag{26}$$

For the last term, we have

$$\text{Tr}\left(m\gamma_5 m\gamma_0\gamma_5\gamma_0\right) = -m^2\text{Tr}\left(\gamma_0\left(\gamma_5\right)^2\gamma_0\right) \tag{27}$$

$$= -m^2\text{Tr}\left(\left(\gamma_0\right)^2\right) \tag{28}$$

$$= -m^2\text{Tr}I \tag{29}$$

$$= -4m^2 \tag{30}$$

Plugging all these results back into 11 we have

$$\sum_{s,s'}\left|\overline{u}_s\left(\mathbf{p}\right)Fv_{s'}\left(\mathbf{p}'\right)\right|^2 = 4p\cdot p'+4m^2 \tag{31}$$

Conservation of momentum requires

$$k^2 = M^2 = \left(p+p'\right)^2 = 2m^2+2p\cdot p' \tag{32}$$

Therefore

$$4p\cdot p' = 2M^2-4m^2 \tag{33}$$

$$\sum_{s,s'}\left|\overline{u}_s\left(\mathbf{p}\right)Fv_{s'}\left(\mathbf{p}'\right)\right|^2 = 2M^2 \tag{34}$$

The remaining calculations in 3 are identical to those done in L&P from equation 7.28 to 7.33. The final result is therefore $\frac{|h'|^2}{2M}$ (from 3) times 34 multiplied by L&P's result for ρ in their equation 7.33, which gives

$$\Gamma = \frac{M\left|h'\right|^2}{8\pi}\sqrt{1-\frac{4m^2}{M^2}} \tag{35}$$

This is equivalent to the result in L&P's equation 7.34 with

$$h^2 \to \frac{\left|h'\right|^2}{\left(1-\frac{4m^2}{M^2}\right)} \tag{36}$$

which is the answer specified in the errata for the textbook.

PINGBACKS

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